# Influence of the size of facets on point focus solar concentrators 

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#### Abstract

It is a common practice in the development of point focus solar concentrators to use multiple identical reflecting facets, as a practical and economic alternative for the design and construction of large systems. This kind of systems behaves in a different manner than continuous paraboloidal concentrators. A theoretical study is carried out to understand the effect of the size of facets and of their optical errors in multiple facet point focus solar concentrating systems. For this purpose, a ray tracing program was developed based on the convolution technique, in which the brightness distribution of the sun and the optical errors of the reflecting surfaces are considered. The study shows that both the peak of concentration and the optimal focal distance of the system strongly depend on the size of the facets, and on their optical errors. These results are useful to help concentrator developers to have a better understanding of the relationship between manufacturing design restrictions and final optical behavior.


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## 1. Introduction

Several different methods have been investigated for the construction of point focus solar concentration systems. These range from segmented parabolas made with metal or glass mirrors, to single surface mirrors that approximate the required geometry, and are fabricated by applying tensions to stretch reflective metallic or polymeric membranes to suitable shapes. A practical solution when very high concentration is required, as for instance in solar furnaces is the formation of the concentrator with individual mirrors of spherical curvature, which can be fabricated in conventional optical workshops. In any case, it is usually assumed that the best focal distance for the system coincides with the focal distance of the continuous parabola one is trying to emulate, or else a simulation is carried out to determine this optimal focal distance for the particular case at hand.

Ray tracing simulations are very effective for the design and optimization of parameters in solar concentration systems [1,2]. They are also used to characterize optical errors in such systems by comparison with experimental results, and for the design of receivers [3-5]. By these techniques it is possible to model large dimensions optical systems like central receivers and faceted concentrators.

[^0]In the present work ray tracing simulations by the convolution method are employed to analyze the effect of the facet size and optical error in the irradiance distribution of point focus concentrators, as well as on the effective value of the ratio between the focal distance and the concentrator width (F/D ratio).

## 2. Methodology

The analysis was carried out by means of a ray tracing program called Tonalli (rising sun in Nahuatl, the Aztec language) [6], developed in Matlab platform in collaboration with CIEMAT, Spain. The program obtains the radiation cone incident in a receiving plane by means of the convolution [7] of a Gaussian distribution of optical errors with the standard solar radiation cone [8], as for instance in the CIRCE2 ray tracing code [9]. There are other techniques for ray tracing to obtain the irradiance distribution on a receiver, like direct ray tracing, in which every considered point on the solar disk, generates a ray that will be reflected on the mirror surface; or the Monte Carlo technique, that use a random set of incidence points. However, the convolution technique is faster for this kind of simulations, because less rays need to be traced.

The simulated system is a faceted concentrator consisting of a paraboloidal structure (frame) where the individual mirrors are attached to their respective positions. However, the curvature of each mirror does not correspond with the part of the frame it covers; It is considered that for ease of fabrication all facets are spherical mirrors; i.e., facets are not fabricated as sections of


Fig. 1. (a) Square facets covering $100 \%$ of the concentrator cross section. (b) Arrangement of the facets once they are located on the paraboloidal frame and tilted to their corresponding angles.
a bigger parabola, but instead as sphere segments with suitable focal length and orientation, as described below.

The vertex of each mirror is located on the paraboloidal frame. And the orientation of the mirror is chosen such as to reflect a ray impinging on its vertex with direction parallel to the axis of the paraboloidal structure, towards the focus of the system. Also, the focal distance of each facet is made equal to the distance from its vertex to the system focus. In this way, all these particular rays impinging at the vertices of the facets are reflected exactly as they would be in a continuous paraboloid. The reflectivity of the mirrors was taken as $100 \%$ for simplicity, but the results may be easily modified if the reflectivity of a given reflective material is considered.

For simplicity, the size and shape of the concentrator were fixed to a $6 \mathrm{~m} \times 6 \mathrm{~m}$ square concentrator. Nevertheless the results are presented in terms of nondimensional quantities, and are therefore not dependent on the area of the concentrator. The concentrator projected area of $6 \mathrm{~m} \times 6 \mathrm{~m}$ is divided into $n \times n$ equal size facets of square shape. The particular selection of square facets should not affect the main ideas discussed here, and is made only for the sake of simplicity.

The square facets would cover the whole cross section of the concentrator if located on the entrance plane, with their individual optical axes normal to it. However, when they are displaced to their respective positions on the paraboloidal frame of the concentrator, and tilted to their respective angles, narrow gaps appear between them, as illustrated in Fig. 1.

It is to be expected that if the number of facets continues increasing, at some point the collecting area of the concentrator would be seriously diminished due to the gaps between the mirrors. Nevertheless, for the number of facets considered in the simulations discussed here, this effect is negligible, as reported in Table 1. The case presented is for a 4.6 m focal distance of the paraboloid. The width of the gaps depends somehow in focal distance, as frames with larger curvatures cause larger gaps.

The parameters varied on the present study were three: the focal distance of the paraboloidal frame, the global optical error of

Table 1
Effective area of the concentrator for different numbers of facets.

| Number of facets | Size of facets | Concentrator effective area |
| :--- | :--- | :--- |
| $6 \times 6=36$ | $1 \times 1 \mathrm{~m}^{2}$ | $34.1783 \mathrm{~m}^{2}$ |
| $12 \times 12=144$ | $0.5 \times 0.5 \mathrm{~m}^{2}$ | $34.1463 \mathrm{~m}^{2}$ |
| $20 \times 20=400$ | $0.3 \times 0.3 \mathrm{~m}^{2}$ | $34.1395 \mathrm{~m}^{2}$ |
| $30 \times 30=900$ | $0.2 \times 0.2 \mathrm{~m}^{2}$ | $34.1374 \mathrm{~m}^{2}$ |

the system, and the number of facets. Meanwhile the concentrator cross sectional area was kept fixed.

The optical error describes non ideal reflection of the sun rays. A Gaussian distribution is assumed for the angular deviation of the reflected rays with respect to the nominal direction of reflection. The latter is given by the specular law of reflection; therefore, the term global optical error refers to the standard deviation of this Gaussian distribution (expressed in milliradians). This error takes into account non specular reflection due to surface microstructure, curvature deviations and misalignment of the facets, as well as tracking errors [10].

In the present work the focal distance of the frame is expressed in terms of its ratio to the width of the concentrator (F/D). Table 2 gives the characteristics of the simulated concentrator.

## 3. Results and discussion

In Figs. 2, 3, and 4, the average irradiance over a flat receiver, sized to collect $90 \%$ of the concentrated power, is shown for different facet numbers and values of the optical error. In all cases the solar beam radiation is taken as $1000 \mathrm{~W} / \mathrm{m}^{2}$, so the concentrated irradiance depicted in the graphs is the same as the average of solar flux concentration. For each system with a given number of facets, the focal distance to width ratio ( $\mathrm{F} / \mathrm{D}$ ) is varied to find the optimal value that maximizes average irradiance. These values correspond to the peak for each of the curves in Figs. 2-4.

In particular, Fig. 2 corresponds to the case without optical errors and shows how the flux concentration increases with the number of facets. This is to be expected, because having the concentrator divided into a larger number of smaller facets provides a more accurate approximation to a continuous paraboloid. On the other

Table 2
Parameters of the simulated concentrator.

| Concentrator size $(\mathrm{D} \times \mathrm{D})$ | $6 \mathrm{~m} \times 6 \mathrm{~m}$ |
| :--- | :--- |
| Frame type | Paraboloid of revolution |
| Focal distance of frame (F/D) | $0.4-1.0$ |
| Facet shape | Square |
| Facet curvature | Spherical |
| Facet focal distance | Equal to distance from facet vertex to the |
|  | focus of the frame |
| Reflectivity of facets | 1 |
| Global optical error | $0-4 \mathrm{mrad}$ |
| Number of facets $(n \times n)$ | $6 \times 6$ to $30 \times 30$ |
| Facets size $(\mathrm{D} / n)$ | $0.20-1.00 \mathrm{~m}$ |
| Receiver type | Flat, without shadowing |
| Receiver size | Variable, to collect $90 \%$ of concentrated power |



Fig. 2. Average irradiance as a function of $F / D$ ratio for different number of facets. Optical error 0 mrad.
hand, the ( $\mathrm{F} / \mathrm{D}$ ) ratio, increases as the number of facets diminishes. For instance, for a concentrator with 114 facets, the optimum is 0.8 times the width of the concentrator, while for an ideal paraboloid it is 0.52 times.

The explanation of the variation of the optimal $\mathrm{F} / \mathrm{D}$ ratio with the number of facets is the following: while radiation impinges normally on the aperture of an ideal paraboloid, in particular the paraboloidal frame of the segmented concentrator, it does not do so on each of the facets. Therefore, the accuracy of the faceted concentrator is reduced by the optical aberrations of the tilted facets, which does not occur in the ideal paraboloid. These aberrations are reduced as the focal distance is increased, because the tilting of the facets is reduced and they now face the incoming radiation at angles closer to normal incidence. Again, as the number of facets increases the concentrator becomes closer to an ideal paraboloid, and the effect of aberrations is also reduced.

In Figs. 3 and 4, results are presented for optical errors of 2 and 3 mrad , respectively. It is clear from these graphs that the optimal focal distance to width ratio for a given number of facets depends on the optical error of the mirrors.

The dependence of the optimal focal distance on optical error is shown in more detail, for the case of $30 \times 30$ facets, in Fig. 5 . The F/D ratio diminishes as the optical error increases. This occurs because a larger optical error implies a reflected solar cone with more angular divergence. Therefore, the image on the receiver increases, and a shorter focal distance is convenient to avoid this. But, on the other hand, a shorter focal distance also implies larger


Fig. 3. Average irradiance as a function of $\mathrm{F} / \mathrm{D}$ ratio for different number of facets. Optical error 2 mrad.


Fig. 4. Average irradiance as a function of $\mathrm{F} / \mathrm{D}$ ratio, for different number of facets. Optical error 4 mrad.


Fig. 5. Average irradiance as a function of $\mathrm{F} / \mathrm{D}$ ratio, for a $30 \times 30$ facets concentration with different optical errors.
aberration effects, which increases the image size. Therefore, the observed optimal value is the result of a trade-off between these two effects, and it moves towards shorter focal distances as the error increases.

Fig. 6 synthesizes the relationship of the optimal F/D ratio with the number of facets and optical error; in general, as discussed before, the optimal focal distance diminishes as either the number of facets or the optical error increase, for a fixed concentrator area.


Fig. 6. Optimal focal distance as a function of the number of facets for different values of optical error.


Fig. 7. Peak concentration as a function of the F/D ratio, for different number of facets. Optical error 0 mrad.


Fig. 8. Peak concentration as a function of the F/D ratio, for different number of facets. Optical error 4 mrad.

A similar dependence with $F / D$ ratio can be observed in the peak of the flux concentration, as shown in Figs. 7 and 8, for 0 and 4 mrad optical errors, respectively. The maximum of peak concentration, for a given number of facets, moves to smaller $\mathrm{F} / \mathrm{D}$ values as the optical error increases. In the case of a continuous paraboloid, a maximum of peak concentration is not observed. This occurs because peak concentration keeps increasing for smaller focal distances, as the reflected radiation cones travel shorter distances. This nevertheless is associated with larger receivers, due to the increased angles subtended by light cones coming from the rim of


Fig. 10. Peak irradiance as a function of the F/D ratio, for a $12 \times 12$ facets concentrator with different optical errors.
the concentrator, and therefore the average concentration start decreasing at some point. In other words, the flux distributions have larger peaks but are more spread on the receiver, as illustrated in Fig. 9.

Again, as in the case of average irradiance, it is observed that the maximum of peak concentration moves to shorter F/D ratios as optical errors increase in multiple facets concentrators. The same happens when the number of facets is increased, due to a closer resemblance of the faceted concentrator to the continuous paraboloid as facets become smaller.

In Fig. 10, the dependence of peak concentration with optical errors and $\mathrm{F} / \mathrm{D}$ ratio is shown in more detail for a $12 \times 12$ facets concentrator.

The results show that, beyond the more or less intuitive result that relates an increased concentration with smaller facet size and optical error, there is a dependence of the optimal focal length with these two variables. In particular, depending on their values, the optimal $\mathrm{F} / \mathrm{D}$ ratio may deviate quite strongly from the well known value of 0.6 for an ideal specular paraboloid. Therefore, both of these parameters must be taken into account when determining the best location of the receiver at the focal zone of the concentrator.

In particular, the results from this study were applied in the optical design of the CIE-UNAM high radiative flux solar furnace [11], which is being built in Temixco, Mexico. In that study the focal distance of the system is chose according to the above results. The radiative flux distribution in a focal plane is evaluated from the size


Fig. 9. Irradiance distributions for different focal distances in continuous paraboloid: (a) 3.6 m (b) 3.0 m (c) 2.4 m (d) 2.0 m .
and number of facets, the global optical error, and the optimal focal distance.

## 4. Conclusions

This work analyzes the effect of the number of facets and optical errors on the average and peak concentrations of faceted paraboloidal reflectors. Facets considered here are not segments of the same paraboloid that forms the frame of the concentrator, but they are spherical mirrors with symmetry of revolution instead, that in many cases can be more easily fabricated. In general, it is found that the larger the number of facets into which the concentrator is divided, the larger the concentrator factors that can be achieved. However, this number is limited in real concentrators by practical considerations. We define the optimal focal distance to width ratio (F/D), for a concentrator with a given area, as that which maximizes the average irradiance (average concentration factor). We find a strong dependence of this optimal value on both the number of facets and the amount of optical error. Therefore, it is not advisable to design a faceted concentrator based only on the theoretical optimal value for a continuous paraboloid, given by $F / D=0.6$, which corresponds to a $45^{\circ}$ rim angle. Instead, every case should be analyzed carefully by means of ray tracing simulations.

In general we find that the optimal $\mathrm{F} / \mathrm{D}$ values can be significantly larger for faceted concentrators with nonzero optical errors than for an ideal continuous paraboloid. This optimal value decreases with the increase of both the number of facets and the amount of optical error. The explanation of the former effect is given in terms of the optical aberrations of the facets.

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