Simulation of the Damage in Reinforced Concrete, by Means of the Embedded Discontinuity Model

Retama V. Jaime[1] **y Ayala M. Gustavo**[2]

[1]Instituto de Ciencias Básicas e Ingeniería, Universidad Autónoma del Estado de Hidalgo
email: JRetamaV@iingen.unam.mx

[2]Instituto de Ingeniería, Universidad Nacional Autónoma de México
Ciudad Universitaria Núm. 3000, Col. Copilco-Universidad
Del. Coyoacán, México D. F., C.P. 04360
e-mail: GayalaM@iingen.unam.mx

Abstract
A variational formulation of the embedded discontinuity model, an its approximation by means of the finite element method, to simulate the damage process in structural elements of reinforced concrete is presented. The effect of the steel bars is introduced through truss elements which are considered to be embedded into a triangle element. Since the reinforced concrete is a composite material, additionally the concrete bulk is modeled by triangle element of constant deformation; for which a cohesive damage model is employed together with the embedded discontinuity approximation. In this work a perfect bold is considered for the interface concrete—steel bars.

Keywords
Finite element, embedded discontinuity, reinforced concrete, damage mechanics, variational formulation.

Introduction

The reinforced concrete has become one of the most employed materials in the construction area of the civil engineering. Its economy, efficiency, and stiffness, make of this an excellent material for the construction of a wide variety of structures, [3]. Unfortunately, due to the complexity involved in its evolution to the collapse, its mechanical behavior is not completely understood. Nowadays different researchers continuous studying its response with experimental tests, [9], and with numerical simulations, [1, 2, 11], the nonlinear behavior of this material, under different conditions which can produce and inelastic response; and eventually can take to the structure to the collapse. Some of the inherent complexities involved in the numerical simulation of the reinforced concrete are:

1. The structural system is three-dimensional and is built of different materials, concrete and steel.
2. The mechanical properties of the composite material, concrete—steel, chance continually due to the development of cracks in the concrete.
3. The bond between concrete and steel materials is an additional effect to take into account in the numerical model.
4. The curve strain—stress, for the concrete, is nonlinear and is function of different state variables.
5. The creep and shirkage have an influence in the deformation of the concrete material; and are dependent of the time.

In the framework of development of new and best numerical models, for the simulation of reinforced concrete elements, [6, 7, 8, 9, 10], one of the most important topic is the experimental test which is essential to establish valid constitutive laws to employed in the numerical modeling by means of finite elements.

Variational formulation

In this paper, the variational formulation presented in the work of Retama, [1], is using to develop a formulation for a solid with an embedded discontinuity, and considering the effect of the reinforcement due to the steel bars. For this consider a solid divides by a discontinuity Γ_d into Ω^- and Ω^+, figure 1, where the relative displacement between both subdomains is defined by the displacements jump $[\![\mathbf{u}]\!]$.

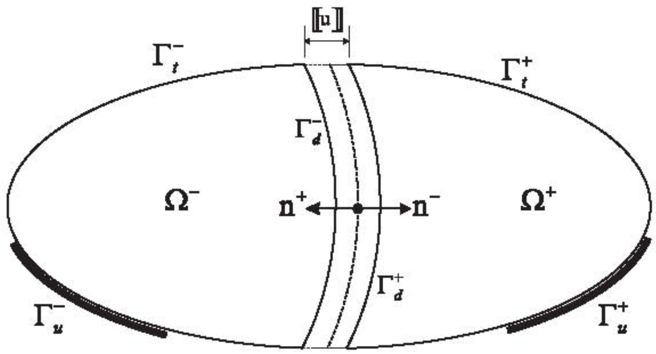

Figure 1. Body with a discontinuity.

The energy functional for this solid, displacement formulation is stated as:

$$\Pi = \int_{\Omega} \left[\Psi(\varepsilon) - \mathbf{u}^T \mathbf{b}_v + \Phi(\mathbf{u}) \right] d\Omega - \int_{\Gamma_t} \mathbf{u}^T t^* \, d\Gamma + \int_{\Gamma_d} \psi\left([\![\mathbf{u}]\!]\right) d\Gamma \tag{1}$$

where $\Psi(\varepsilon)$ is the strain energy function, $\Phi(\mathbf{u})$ is a functions which considers the dissipation energy in the interface between the concrete and the steel bars and $\psi\left([\![\mathbf{u}]\!]\right)$ is the elastic free energy density defined on the discontinuity boundary.

Cohesive damage model

For modeling the evolution of material damage where mode I type failure is dominant, a cohesive crack model is used, [12]. Micro—cracking and plastic flow around a macroscopic crack tip are modeled as equivalent traction forces on crack faces, figure 2.

In this model the inelastic response is governed by two key material parameters: the tensile strength σ_{t_0} and the fracture energy G_F. A discontinuity is introduced when the maximum principal stress exceeds the tensile strength of the material, Rankine criterion.

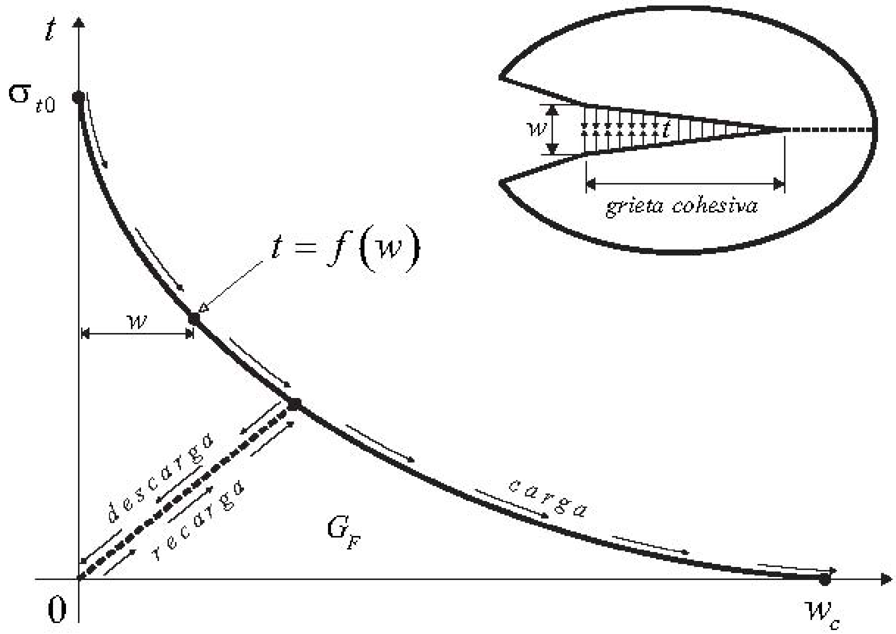

Figure 2. Capturing a micro—cracking zone into a cohesive surface.

The function f , which defines the state of the discontinuity, loading/unloading, is defined as:

$$f = \langle [\![u]\!]_n \rangle - \kappa \leq 0 \tag{2}$$

where $[\![u]\!]_n$ is the normal component of the displacement jump. The symbols $\langle \bullet \rangle$ are the McAuley brackets denoting that only the positive part of $[\![u]\!]_n$ is considered. κ is an internal scalar variable equal to the highest value of $[\![u]\!]_n$.

The constitutive damage law, traction—jump relation, is defined by:

$$t_n = \sigma_{t_0} \left[1 - \frac{\kappa}{w_c} \right]^n \tag{3}$$

where w_c is the maximum normal jump allowed on the discontinuity and n is a constant which defines the shape of the softening curve, figure 2.

From equation 3, the constitutive tensor $\mathbf{T}$ for the discontinuity boundary is recovered as:

$$\mathbf{T} = \frac{\partial t}{\partial \kappa} = -\frac{n\,\sigma_{t_0}}{w_c} \left[1 - \frac{\kappa}{w_c} \right]^{(n-1)} \tag{4}$$

To obtain a consistent energy dissipation rate, the constitutive law is related with the fracture energy in the next form:

$$G_F = \int_0^{w_c} t_n \, d\kappa \tag{5}$$

Finite element approximation

To approximate the displacement field, the finite element method is used. To begin with, it is considered that the steel bars are localized in the solid element, concrete material, domain as it is shown in figure 3.

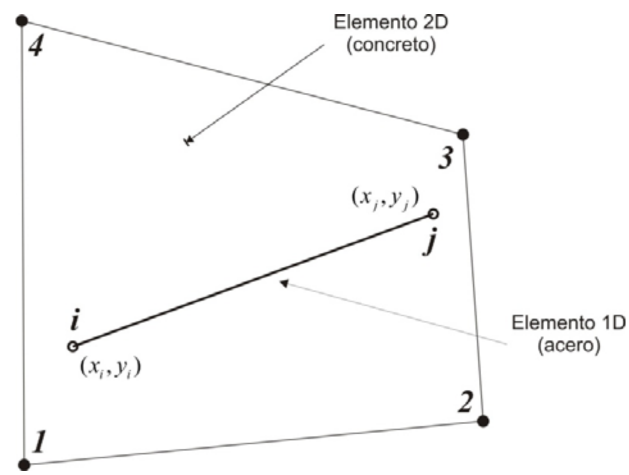

Figura 3. A rectangular reinforced concrete element.

Unlike other reinforced concrete elements, here it is possible that the steel bars crossed the concrete element. The advantage of this is that not structures meshes are necessary to accommodate the effect of the reinforcement.

The stiffness matrix of the reinforced concrete element is written as:

$$k_e = k_1 + k_2 \tag{6}$$

where k_1 is a matrix as for an standard finite element, bi-dimensional, but considering a discontinuity in its domain, the condensed for of this matrix is stated as

$$k_1 = k_{\hat{u}\hat{u}} - k_{\hat{u}\tilde{u}} \left[k_{\tilde{u}\tilde{u}} \right]^{-1} k_{\hat{u}\tilde{u}}^T \tag{7}$$

with

$$k_{\hat{u}\hat{u}} = \int_{\Omega} B^T \mathbf{C} B \, d\Omega \tag{8}$$

$$k_{\hat{u}\tilde{u}} = \int_{\Omega} B^T \mathbf{C} B_c \, d\Omega \tag{9}$$

$$k_{\tilde{u}\tilde{u}} = \int_{\Omega} B_c^T \mathbf{C} B_c \, d\Omega + \int_{\Gamma_d} \mathbf{T} \, d\Gamma \tag{10}$$

and the matrix k_2 takes into account the contribution of the reinforced bars through consider a truss element, one-dimensional, inside a the bi-dimensional element, figure 3, and is defined by

$$k_2 = R_n^T R_\alpha^T k_t R_\alpha R_n \tag{11}$$

with

$$R_\alpha = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 & 0 \\ 0 & 0 & \cos\alpha & \sin\alpha \end{bmatrix} \tag{12}$$

$$R_n = \begin{bmatrix} n1^i & 0 & n2^i & 0 & n3^i & 0 \\ 0 & n1^i & 0 & n2^i & 0 & n3^i \\ n1^j & 0 & n2^j & 0 & n3^j & 0 \\ 0 & n1^j & 0 & n2^j & 0 & n3^j \end{bmatrix} \tag{13}$$

and the matrix k_t corresponds to the truss elements one. The matrix R_α is used to rotate the truss element matrix into global coordinates and R_n is a matrix which transmits the effect of the rotated truss matrix to the bi-dimensional concrete element. $n1^i$, $n2^i$ and $n3^i$ are the bi-dimensional shape functions evaluated in the node i of the truss element whereas $n1^j$, $n2^j$ and $n3^j$ means the same but for the node j. For a detailed description of the variational formulation, the reader is addressed to the work of Retama, [1].

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