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Consistent symmetric formulation of the Enhanced Embedded Discontinuity Method

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The need of accurately modeling phenomena that present strong discontinuities in the fields of the state variables arises in many fields of the mechanics of materials and structures. Either vector (displacement, flow) or scalar (temperature, ion concentration) fields can present discontinuities. The Strong Discontinuity Approach (SDA) has proven effective in treating many of these cases; its basic idea is to introduce an enhancement in the interpolation of the relevant field, which bears the required discontinuity only where it appears, so that the model of the main field is not affected by the enhancement.

The idea is shared between methods that enrich the nodal variables (XFEM) and methods that use elements with embedded discontinuities (EED). The latter appear very convenient since the additional degrees of freedom enter the equations only locally, and can be treated in the same way as inelastic strains are [2].

There exist a large number of formulations for EED [3, 4] that differ mainly in the numerical implementation of the method, since the kinematic assumptions are basically the same as those originally proposed by Simo [5]. Furthermore, it appears that most of the formulation have been developed within the framework of constant stress elements, or at least for elements with constant enhanced strains (however, see [6, 7]). Finally, the case of dissipating continua has not yet been addressed completely, partially because of a deficiency in the rigorous variational setting of the problem.

The paper preliminary defines a variational formulation for a continuum medium Ω, figure 1, characterized by an elastic-plastic damaging behavior in which the growth of interfaces S takes places. The activation of the interface is ruled by a specific dissipation function, based on a cohesive fracture like criterion. The mixed multi-fields Hu-Washizu functional Π^{HW} has among its arguments the displacement jump $[\![\mathbf{u}]\!]$ and the conjugated traction on the discontinuity surface S, $\mathbf{t}_S$, [1]. Attention is then devoted to the kinematic of the enhancement displacement field. As explained in the paper by Jirasek [8], using the standard interpolation of the enhanced displacement field a Ritz-Galerkin procedure produces a symmetric formulation, but violates the equilibrium condition on the interface, while a statically consistent formulation derives from a Petrov-Galerkin approach, thus leading to a non symmetric problem. How-

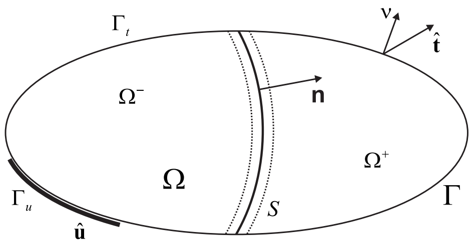

Figure 1: Continuum crossed by a discontinuity

ever this interpolation presents some problems: first of all, it violates Dirichlet boundary conditions in the domain where the enhancement is used. This aspect is dealt with in a parent paper presented in this same conference, where it is shown that a rigorous definition of the enhancement field that satisfies the Dirichlet BC's also leads to a weak satisfaction of the traction continuity condition on the interface.

The conditions presented in [9], however, guarantee weak equilibrium of the interface in a global sense, but generally not element by element. In this paper it is presented an alternative formulation of the enhancement displacement field that allows to use a symmetric Ritz-Galerkin approach, and that satisfies traction continuity within the element. The formulation, as a consequence, is specific for each element used in the discretisation.

The paper will compare the performance of the newly proposed enhanced element with standard enriched elements and with smeared crack elements. Particularly, the consequences on the strain field will be discussed.

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