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Modelling of damage in solids by weak and discrete embedded discontinuities approximations

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ABSTRACT

A comparative study on the modelling of damage in quasi-brittle materials is presented. The embedded discontinuities formulation is used in its weak and discrete approximations. A quasi-static load condition is considered. It is assumed that crack initiation occurs according to *Mode I* of failure with its posterior evolution into a mixed mode.

Damage is introduced as a weakness band, weak discontinuity, where nonlinear effects are smeared with a band width different from zero, $h \neq 0$. Continuum constitutives strain-stress laws are used in the whole domain [4, 5]. If band width is equal to zero, $h = 0$, a true crack is introduced, discrete discontinuity. In this case two differents constitutives relationships are used; a continuum constitutive law, in the bulk, and the other traction-jump relationship along the discontinuity boundary [1, 6, 9].

The variational formulation used here, corresponds to the *embedded discontinuities* formulation developed by Juárez and Ayala [2, 3]. In this formulation, cracks are introduced in the kinematic of the displacement field [7, 8]. Energy functionals, corresponding to: displacement and strain-displacement formulations, for the weak discontinuities approximation are:

$$\Pi(u) = \int_{\Omega} [W(\bar{\varepsilon}) - \mathbf{b} \cdot \mathbf{u}] \, d\Omega - \int_{\Gamma_\sigma} \mathbf{t}^* \cdot \mathbf{u} \, d\Gamma + h \int_{\mathrm{S}} [W(\tilde{\varepsilon}) - \mathbf{b} \cdot \mathbf{u}] \, d\mathrm{S} \tag{1}$$

$$\begin{aligned} \Pi(u, \varepsilon) = & \int_{\Omega} [\bar{\boldsymbol{\varepsilon}}^u \cdot \mathbf{C} \cdot \bar{\boldsymbol{\varepsilon}} - W(\bar{\varepsilon}) - \mathbf{b} \cdot \mathbf{u}] d\Omega - \int_{\Gamma_\sigma} \mathbf{t}^* \cdot \mathbf{u} d\Gamma \\ & + h \int_{\mathrm{S}} [\tilde{\boldsymbol{\varepsilon}}^u \cdot \mathbf{C} \cdot \tilde{\boldsymbol{\varepsilon}} - W(\tilde{\varepsilon}) - \mathbf{b} \cdot \mathbf{u}] d\mathrm{S} \end{aligned} \tag{2}$$

In the same fashion the energy functionals, for both formulations, corresponding to the discrete approximations may be written as:

$$\Pi(u) = \int_{\Omega} [W(\bar{\varepsilon}) - \mathbf{b} \cdot \mathbf{u}] \, d\Omega - \int_{\Gamma_\sigma} \mathbf{t}^* \cdot \mathbf{u} \, d\Gamma + \frac{1}{2} \int_{\mathrm{S}} [|\mathbf{u}|] \cdot \mathbf{T} \cdot [|\mathbf{u}|] \; d\mathrm{S} \tag{3}$$

$$\begin{aligned} \Pi(u, \varepsilon) = &\int_{\Omega} [\bar{\boldsymbol{\varepsilon}}^u \cdot \mathbf{C} \cdot \bar{\boldsymbol{\varepsilon}} - W(\bar{\varepsilon}) - \mathbf{b} \cdot \mathbf{u}] d\Omega - \int_{\Gamma_\sigma} \mathbf{t}^* \cdot \mathbf{u} d\Gamma \\ &+ \frac{1}{2} \int_{\mathrm{S}} [|\mathbf{u}|] \cdot \mathbf{T} \cdot [|\mathbf{u}|] \; d\mathrm{S} \end{aligned} \tag{4}$$

Two different finite element formulations, in the numerical implementation, are investigated. One is the displacement formulation and the other is a mixed where two fields, displacements and strains, are interpoled as independent.

Numerical examples are presented to show the advantages and disadvantages of each implemented formulation. At the end of the paper, conclusions and some relevant comments, about the numerical damage modelling by the embedded discontinuity formulation, are discussed.

References

[1] J. Alfaiate, G. N. Wells, and L. J. Sluys. On the use of embedded discontinuity elements with crack path continuity for mode-I and mixed-mode fracture. *Engng. Fracture Mech.*, 69(6):661–686, 2002.

[2] G. Juárez. *Numerical modelling of fracture problems in solids by embedded discontinuities*. PhD thesis, Universidad Nacional Autónoma de México, México, 2006. In Spanish.

[3] G. Juárez and G. Ayala. Variational formulation of the material failure process in solids by embedded discontinuities model. *Numerical Methods for Partial Differential Equations*, 25:26–62, 2009.

[4] J. Lemaitre. *A Course on Damage Mechanics*. Springer-Verlag, Germany, 1996.

[5] J. Lemaitre and R. Desmorat. *Engineering Damage Mechanics*. Springer-Verlag, Germany, 2005.

[6] Y. N. Li and Z. P. Bažant. Eigenvalue analysis of size effect for cohesive crack model. *Int. J. Fracture*, 66:213–226, 1994.

[7] J. Oliver. Modelling strong discontinuities in solid mechanics via strain softening constitutive equations. Part 1: Fundamentals. *Int. J. Numer. Meth. Engrg.*, 39(21):3575–3600, 1996.

[8] J. Oliver. Modelling strong discontinuities in solid mechanics via strain softening constitutive equations. Part 2: Numerical simulation. *Int. J. Numer. Meth. Engrg.*, 39(21):3601–3623, 1996.

[9] G. N. Wells and L. J. Sluys. Three–dimensional embedded discontinuity model for brittle fracture. *Int. J. Solids & Structures*, 38(5):897–913, 2001.