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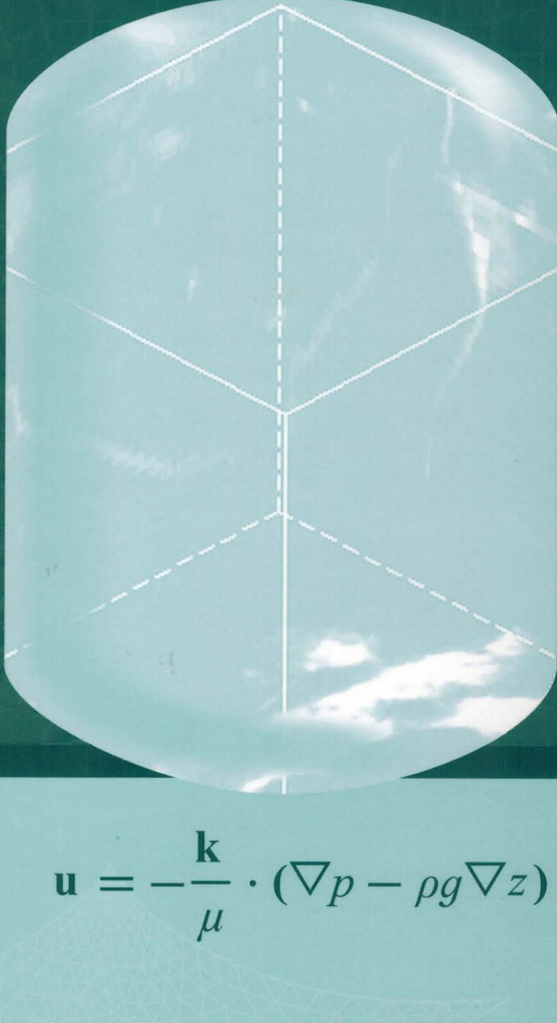

$$\mathbf{u} = -\frac{\mathbf{k}}{\mu} \cdot (\nabla p - \rho g \nabla z)$$

Mathematical and Numerical Modeling in Porous Media: Applications in Geosciences

Martín A. Díaz Viera, Pratap N. Sahay, Manuel Coronado and Arturo Ortiz Tapia
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Section 3: Statistical and stochastic characterization

CHAPTER 10

Trivariate nonparametric dependence modeling of petrophysical properties

A. Erdely, M.A. Díaz-Viera & V. Hernández-Maldonado

10.1 INTRODUCTION

Assessment of formation permeability is a complex and challenging problem that plays a key role in oil reservoir forecasts and optimized reservoir management. Generally, permeability evaluation is performed using porosity-permeability relationships obtained by integrated analysis of various petrophysical parameters from cores and well logs. In carbonate double-porosity formations with complex microstructure of pore space this problem becomes more difficult because the permeability usually does not depend on the total porosity, but on classes of porosity, such as vuggular and fracture porosity (secondary porosity). Even more, in such cases permeability is directly related to the connectivity structure of the pore system. This fact makes permeability prediction a challenging task.

Dependence relationships among petrophysical random variables, such as permeability and porosity, are usually nonlinear and complex, and therefore those statistical tools that rely on assumptions of linearity and/or normality are not suitable in this case. The use of copulas for modeling petrophysical dependencies is not new, see Díaz-Viera & Casar-González (2005) where *t*-copulas have been used for this purpose. But expecting a single copula family to be able to model any kind of bivariate dependency seems to be still too restrictive, at least for the petrophysical variables under consideration in this work. Therefore, it has been adopted a nonparametric approach by the use of the Bernstein copula, see Sancetta & Satchell (2004) and Sancetta (2007).

10.1.1 *The problem of modeling the complex dependence pattern between porosity and permeability in carbonate formations*

According to Balan (1995) by far the most used permeability predictor is the porosity-permeability relationship. It has long been assumed that most reservoir rocks show a reasonably linear relationship between these parameters in a semi-log scale, which allows for the estimation of permeability when a porosity profile is available. This normally requires a calibration data set that is represented by one or more key wells where comprehensive information is available in terms of core and log data. This calibration data set is used to build the predictor and to test the reliability of the results.

The regression approach, using statistical instead of deterministic formalism, tries to predict a conditional average, or expectation of permeability, corresponding to a given set of parameters. A different predictive equation must be established for each new area or new field. The main drawback of traditional regression methods is that the complex variability of data may not be effectively captured just in terms of variance or standard deviation (which may not even exist), and therefore the predicted permeability profile will be ineffective in reproducing extreme values.

What is important to note in this case is that the predicted permeability profile will be effective in estimating the average characteristics of the true profile, but will be ineffective in estimating the extreme values. These extreme values, from a fluid flow point of view, are the most important parts of the distribution, since they may represent either high permeability streaks or impermeable barriers.

Reservoir rocks show a wide spectrum of porosity-permeability relationships. In some formations, like for example homogeneous clastic rocks, these relationships show very low dispersion and can be reasonably used for prediction purposes. In other cases, as it is frequently for carbonates, this relationship is very loose and does not allow any safe regression, under traditional statistical tools.

On the other hand, model-free function estimators like artificial neural networks are very flexible tools for recognizing and reproducing the pattern of permeability distribution, but require a time consuming "learning" process which strongly depends on the amount and quality of available data.

A competitive and more systematic method for predicting permeability may be achieved by applying stochastic joint simulations, in which the correct specification of dependence pattern in the bivariate porosity-permeability distribution is crucial. According to Deutsch (1994) this approach basically consisted on an annealing geostatistical porosity-permeability cosimulation using their empirical joint distribution. A modification of the previous methodology was proposed by Díaz-Viera & Casar-González (2005) where the basic idea was to apply a t-copula bivariate distribution instead of the empirical one, in which the permeability-porosity observed dependence pattern is specified through a rank correlation measure such as Kendall's tau or Spearman's rho.

10.1.2 *Trivariate copula and random variables dependence*

According to Sklar's Theorem, see Sklar (1959), the underlying *trivariate copula* associated to a trivariate random vector (X, Y, Z) represents a functional link between the joint probability distribution D and the univariate marginal distributions F, G and H, respectively:

$$D(x, y, z) = C(F(x), G(y), H(z)) \tag{10.1}$$

for all x, y, z in the extended real numbers system, where $C : [0, 1]^3 \to [0, 1]$ is unique whenever X, Y and Z are continuous random variables. Therefore, all the information about the dependence between random variables is contained in their corresponding copula. Several properties may be derived for copulas, see Schweizer & Sklar (1983) and Nelsen (2006), and among there is an immediate corollary from Sklar's Theorem: X, Y and Z are independent continuous random variables if and only if their underlying copula is $\Pi(u, v, w) = uvw$. Another interesting property is the fact that copulas are invariant under strictly increasing transformations of the random variables: the copula for (X, Y, Z) is the same than the one for $(g_1(X), g_2(Y), g_3(Z))$, where the g_i functions are strictly increasing and well defined in the range of the corresponding random variables.

Let $\mathcal{S} := \{(x_1, y_1, z_1), \ldots, (x_n, y_n, z_n)\}$ be observations of a random vector (X, Y, Z). Empirical estimates may be obtained for the marginal distributions of X, Y and Z by means of

$$F_n(x) = \frac{1}{n}\sum_{m=1}^{n} \mathbb{I}\{x_m \leq x\}, \;\; G_n(y) = \frac{1}{n}\sum_{m=1}^{n} \mathbb{I}\{y_m \leq y\}, \;\; H_n(z) = \frac{1}{n}\sum_{m=1}^{n} \mathbb{I}\{z_m \leq z\}, \tag{10.2}$$

where $\mathbb{I}$ stands for an indicator function which takes a value equal to 1 whenever its argument is true, and 0 otherwise. It is well-known, see for example Billingsley (1995), that the empirical distribution F_n is a consistent estimator of F, that is, $F_n(t)$ converges almost surely to $F(t)$ as $n \to \infty$, for all t.

Similarly, from Deheuvels (1979) we have the *empirical copula,* a function C_n with domain $\{i/n : i = 0, 1, \ldots, n\}^3$ defined as

$$C_n\left(\frac{i}{n}, \frac{j}{n}, \frac{k}{n}\right) = \frac{1}{n}\sum_{m=1}^{n} \mathbb{I}\{rank(x_m) \leq i,\; rank(y_m) \leq j,\; rank(z_m) \leq k\} \tag{10.3}$$

and its convergence to the true copula C has also been proved, see Fermanian et al. (2004). The empirical copula is not a copula, since it is only defined on a finite grid, not in the whole unit cube $[0, 1]^3$, but by Sklar's Theorem C_n may be extended to a copula.

10.2 TRIVARIATE DATA MODELING

A copula-based nonparametric approach is proposed to model the relationship between permeability, porosity and shear wave velocity (S-waves) of the double porosity carbonate formations of a South Florida Aquifer in the western Hillsboro Basin of Palm Beach County, Florida.

The characterization of this aquifer for the borehole and field scales is given in Parra et al. (2001) and Parra & Hackert (2002), and a hydrogeological situation is described by Bennett et al. (2002). The interpretation of the borehole data and determination of the matrix and secondary porosity and secondary-pore types (shapes of spheroids approximating secondary pores) were presented by Kazatchenko et al. (2006a), where to determine the pore microstructure of aquifer carbonate formations the authors applied the petrophysical inversion technique that consists in minimizing a cost function that includes the sum of weighted square differences between the experimentally measured and theoretically calculated logs as in Kazatchenko et al. (2004).

In this case the following well logs were used for joint simultaneous inversion as input data: resistivity log, transit times of the P- and S-waves (acoustic log), total porosity (neutron log), and formation density (density log). To calculate the theoretical acoustic and resistivity logs the double-porosity model for describing carbonate formations was applied: Kazatchenko et al. (2006b).

This model treats carbonate rocks as a composite material that consists of a homogeneous isotropic matrix (solid skeleton and matrix pore system) where the secondary pores of different shapes are embedded. The secondary pores were approximated by spheroids with variable aspect ratios to represent different secondary porosity types: vugs (close-to-sphere shapes), quasi-vugs (oblate spheroids), channels (prolate spheroids), and microfractures (flattened spheroids). For computing the effective properties the symmetrical self-consistent method of the effective medium approximation was used.

In this paper it has been used the results of inversion obtained by Kazatchenko et al. (2006a) for carbonate formations of South Florida Aquifer that includes the following petrophysical characteristics: matrix porosity, secondary vugular and crack porosities. It should be noted that the secondary-porosity system of this formation has complex microstructure and corresponds to a model with two types of pore shapes: cracks (flattened ellipsoids) with the overall porosity of 2% and vugs (close to sphere) with the porosity variations in the range of 10–30%. Such a secondary-porosity model can be interpreted as the interconnection of microfractures and channels vugular formation.

The relative vugular porosities (PHIV), that is vugular porosity divided by matrix porosity, is modeled as an absolutely continuous random variable X with unknown marginal distribution function F, shear wave velocity (VS meas = velocity of S-waves measured) as an absolutely continuous random variable Y with unknown marginal distribution function G, and permeability (K) as an absolutely continuous random variable Z with unknown marginal distribution function H. Trivariate observations from the random vector (X, Y, Z) are obtained from Kazatchenko et al. (2006a). For continuous random variables, the use of the empirical distribution function estimates Eq. (10.2) is not appropriate since F_n is a step function, and therefore discontinuous, so a smoothing technique is needed: a smooth estimation of the marginal quantile function $Q(u) = F^{-1}(u) = \inf\{x : F(x) \geq u\}$, $0 \leq u \leq 1$, which is possible by means of Bernstein polynomials as in Muñoz-Pérez & Fernández-Palacín (1987):

$$\widetilde{Q}_n(u) = \sum_{m=0}^{n} \frac{1}{2}(x_m + x_{m+1})\binom{n}{m} u^m (1-u)^{n-m}, \tag{10.4}$$

and the analogous case for marginals G and H in terms of values y_m and z_m obtaining $\widetilde{R}_n(v)$ and $\widetilde{S}_n(w)$, respectively. For a smooth estimation of the underlying copula it has been used the Bernstein copula as in Sancetta & Satchell (2004) and Sancetta (2007):

$$\widetilde{C}(u,v,w) = \sum_{i=0}^{n}\sum_{j=0}^{n}\sum_{k=0}^{n} C_n\left(\frac{i}{n},\frac{j}{n},\frac{k}{n}\right)\binom{n}{i}u^i(1-u)^{n-i}\binom{n}{j}v^j(1-v)^{n-j}\binom{n}{k}w^j(1-w)^{n-k} \tag{10.5}$$

for every (u, v, w) in the unit cube $[0, 1]^3$, and where C_n is as defined in Eq. (10.3).

10.3 NONPARAMETRIC REGRESSION

The main objetive in regression models is to explain/predict a random variable of interest (permeability) in terms of some other explanatory variables (relative vugular porosity and shear wave velocity, for example). A nonparametric approach is recommended when the data does not exhibit a "nice" behavior that might suggest some parametric models to be fitted, and/or when the assumptions of parametric candidates are too strong to be considered realistic in a particular case. Since the copula approach – parametric or not – leads to the estimation of the joint probability distribution of the involved variables, from this last one it is possible to obtain/estimate the conditional distribution of the variable of interest given certain values of the explanatory variables, and therefore point and interval estimates may be derived by means of a regression curve or regression surface, instead of imposing a functional form to such curve or surface as it happens, for example, in classical multiple linear regression, and which may happen to be unrealistic. But a no-free-lunch principle appplies: under the copula approach, intensive computational issues arise if many variables are involved.

Given multivariate data, it is common to start choosing as explanatory variables those who exhibit higher dependence with the variable that is to be explained. Pearson's linear correlation coefficient has widely been used for this purpose, but unfortunately it has many flaws that may induce to misleading conclusions, see Embrechts et al. (1999, 2003) for a discussion why it should not be considered as a dependence measure for general purposes, specially when normality and/or moment existence and/or linear dependence are unrealistic assumptions, which happens to be the case of the data under consideration. Therefore, dependence has been measured in terms of the dependence index Φ proposed by Hoeffding (1940), which satisfies all desirable properties for a dependence measure for continuous random variables, see for example Nelsen (2006).

From all the possible explanatory variables for permeability (K) in Kazatchenko et al. (2006a), for the first explanatory random variable it was chosen relative vugular porosity (PHIV) since it exhibited the highest dependence $\Phi(\text{PHIV}, \text{K}) = 0.71$ (on a $[0, 1]$ scale). In choosing a second explanatory random variable it is preferred, in addition to have a high dependence with permeability, the lowest possible dependence with the first explanatory variable (PHIV), otherwise it would mean that it is quite similar to it and it will add no significant information to what the first one already can provide. Under this criteria, the second best choice was the share wave velocity (VS meas), with $\Phi(\text{VS meas}, \text{K}) = 0.60$ and $\Phi(\text{PHIV}, \text{VS meas}) = 0.55$. At this point, no more explanatory variables are considered since intensive computational issues arise when dealing with a 4-dimensional Bernstein copula. Hence, we model a trivariate random vector $(X, Y, Z) = (\text{PHIV}, \text{VS meas}, \text{K})$ under a nonparametric copula approach, and by conditioning we may obtain two regression curves (K given PHIV and K given VS meas) and one regression surface (K given PHIV and VS meas jointly).

For a value x in the range of the random variable X and $0 < \alpha < 1$ let $z = \beta_\alpha(x)$ denote a solution to the equation $\mathbb{P}(Z \leq z | X = x) = \alpha$. Then the graph of $z = \beta_\alpha(x)$ is the *α-quantile regression curve* of Z conditional on $X = x$. The particular case $\alpha = 0.5$ is the *median regression curve*. It has been proved in Nelsen (2006) that

$$\mathbb{P}(Z \leq z | X = x) = c_u(w)\big|_{u=F(x),w=H(z)}, \tag{10.6}$$

where

$$c_u(w) = \frac{\partial \widetilde{C}(u, w)}{\partial u}, \tag{10.7}$$

and $\widetilde{C}(u, w) = \widetilde{C}(u, 1, w)$, the bivariate case of Eq. (10.5). The above result (10.6) leads to the following algorithm in Nelsen (2006) to obtain the α-quantile regression curve of Z conditional on $X = x$.

ALGORITHM 1

1. Set $c_u(w) = \alpha$.
2. Solve for the regression curve $w = \gamma_\alpha(u)$.
3. Replace u by $\widetilde{Q}_n^{-1}(x)$ and w by $\widetilde{S}_n^{-1}(z)$, see Eq. (10.4).
4. Solve for the regression curve $z = \beta_\alpha(x)$.

In Figure 10.1 it is shown a scatter plot of relative vugular porosity (PHIV) versus permeability (K), along with three regression curves: $\alpha = 0.5$ (median) which represents a point estimate of K given PHIV values, and with $\alpha = 0.1, 0.9$ which represent 80% probability bands for the point estimates. In Figure 10.2 it is shown the log-scale values of K in spatial form (in terms of depth) and point estimates of log K given spatial values of PHIV. We have the analogous cases for shear wave velocity (VS meas) in Figures 10.3 and 10.4.

For a value x in the range of the random variable X, a value y in the range of the random variable Y and $0 < \alpha < 1$ let $z = \beta_\alpha(x, y)$ denote a solution to the equation $\mathbb{P}(Z \leq z \mid X = x, Y = y) = \alpha$. Then the graph of $z = \beta_\alpha(x, y)$ is the α-quantile regression surface of Z conditional on $X = x$ and $Y = y$. Let

$$c_{uv}(w) = \frac{\partial \widetilde{C}(u, v, w)}{\partial u \partial v} \bigg/ \frac{\partial \widetilde{C}(u, v, 1)}{\partial u \partial v} \tag{10.8}$$

where $\widetilde{C}$ is obtained by Eq. (10.5). Then:

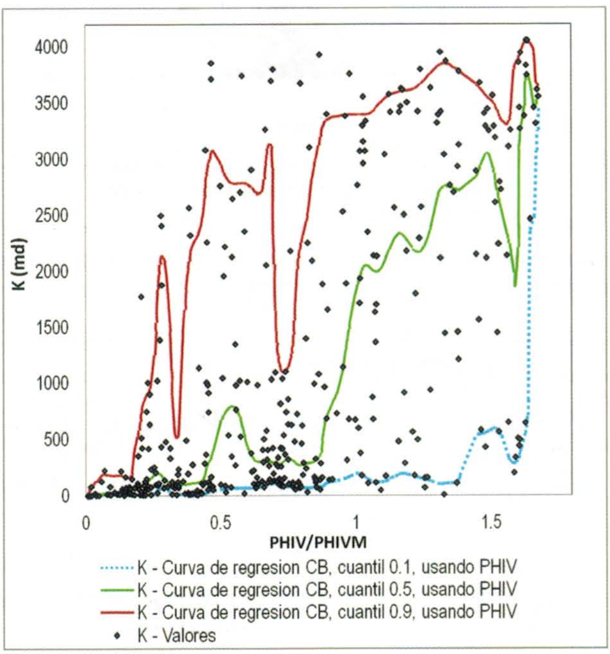

Figure 10.1 Scatterplot of relative vugular porosity (PHIV) and permeability (K) data, with $\alpha = 0.1, 0.5, 0.9$ quantile regression curves.

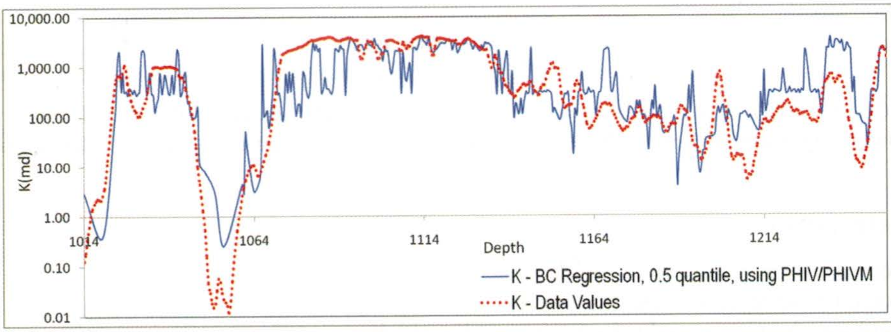

Figure 10.2 Log-scale permeability (K) spatial data (in terms of depth) and spatial median regression curve given relative vugular porosity (PHIV).

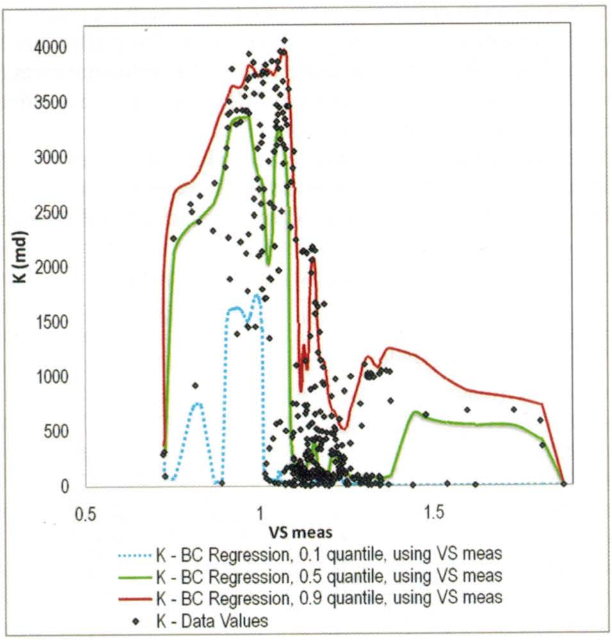

Figure 10.3 Scatterplot of shear wave velocity (VS meas) and permeability (K) data, with $\alpha = 0.1, 0.5, 0.9$ quantile regression curves.

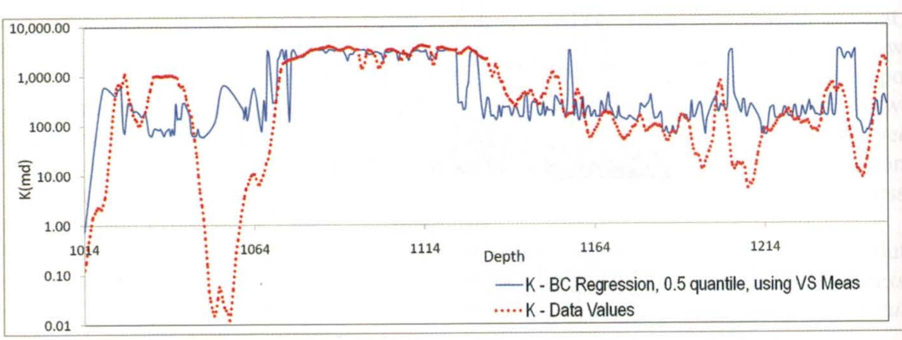

Figure 10.4 Log-scale permeability (K) spatial data (in terms of depth) and spatial median regression curve given shear wave velocity (VS meas).

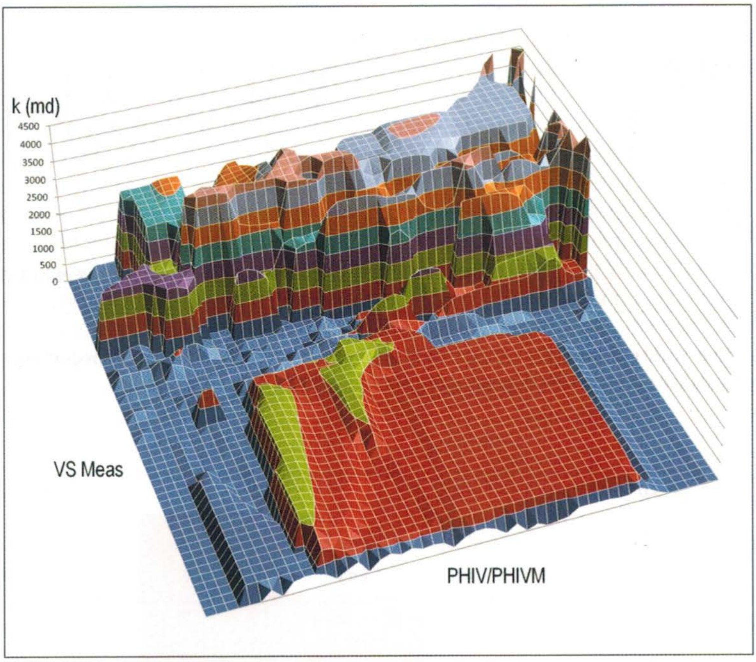

Figure 10.5 Median regression surface for permeability (K) given relative vugular porosity (PHIV) and shear wave velocity (VS meas).

ALGORITHM 2

1. Set $c_{uv}(w) = \alpha$.
2. Solve for the regression surface $w = \gamma_\alpha(u, v)$.
3. Replace u by $\widetilde{Q}_n^{-1}(x)$, v by $\widetilde{R}_n^{-1}(y)$, and w by $\widetilde{S}_n^{-1}(z)$, see Eq. (10.4).
4. Solve for the regression surface $z = \beta_\alpha(x, y)$.

In Figure 10.5 it is shown the median regression surface for permeability (K) given relative vugular porosity (PHIV) and shear wave velocity (VS meas), and in Figure 10.6 the log-scale values of K in spatial form (in terms of depth) and point estimates of log K given spatial values of PHIV and VS meas.

As a descriptive measure of the goodness of fit of predicted values of log-permeability (log K) given values of the explanatory variables PHIV and VS meas, the mean squared error (MSE) has been calculated in each case:

$$\text{MSE}\,(\log \text{K} \mid \text{PHIV}) = \frac{1}{n}\sum_{m=1}^{n}[z_m - \beta_{0.5}(x_m)]^2$$

$$\text{MSE}\,(\log \text{K} \mid \text{VS meas}) = \frac{1}{n}\sum_{m=1}^{n}[z_m - \beta_{0.5}(y_m)]^2$$

$$\text{MSE}\,(\log \text{K} \mid \text{PHIV, VS meas}) = \frac{1}{n}\sum_{m=1}^{n}[z_m - \beta_{0.5}(x_m, y_m)]^2$$

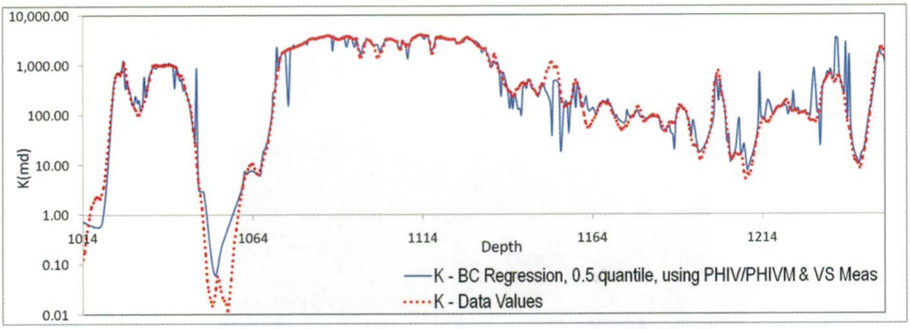

Figure 10.6 Log-scale permeability (K) spatial data (in terms of depth) and spatial median regression curve given relative vugular porosity (PHIV) and shear wave velocity (VS meas).

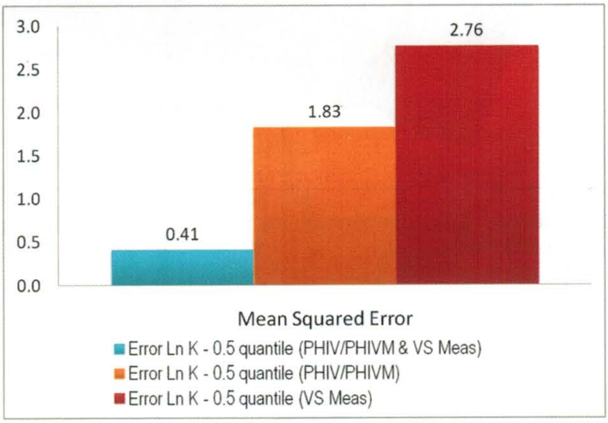

Figure 10.7 Mean squared error of spatial regression curves for permeability (K) given: a) relative vugular porosity (PHIV) and shear wave velocity (VS meas) jointly, b) relative vugular porosity alone, c) shear wave velocity alone.

where the lower the MSE value, the better the goodness of fit. As expected, in Figure 10.7 we have that the spatially predicted log K values have the lowest MSE when using both explanatory variables than each one alone. Also, the MSE of PHIV alone is better than VS meas alone, which was also expected since PHIV has a higher dependence value with K ($\Phi = 0.71$) that VS meas with K ($\Phi = 0.60$).

10.4 CONCLUSIONS

From a methodological point of view, this approach provides a very flexible statistical research tool to investigate the existing complex dependence relationships of petrophysical properties such as porosity and permeability, without imposing strong assumptions of linearity or log-linearity, and/or normality when modeling them as random variables, not even the existence of first or second moments of the variables involved. The only assumption has been the random variables to be jointly absolutely continuous, and thereafter the data is allowed to speak by itself about the dependence structure.

The methodology used in this work has the main advantage of being a straightforward way to perform nonparametric quantile regression, which is useful in obtaining conditional point and interval estimates for Z given $X = x$ and $Y = y$ without imposing or assuming functional relationships among the variables involved.

All the information about the dependence structure is contained in the underlying copula, and its estimation is being used, instead of the extreme information reduction that is done by the use of numerical measures such as the linear correlation coefficient, which under the presence of nonlinear dependence may become useless and/or quite misleading, see Embrechts et al. (1999, 2003). The nonparametric regression obtained is useful to confirm or to question prior ideas about relationships among variables, or even in proposing an apropriate model to explain such relations.

In relation with the geostatistical applications this method opens a promising line of research to model in a nonparametric fashion the intrinsic spatial dependence of random functions overcoming the restriction imposed by a linear co-regionalization models. This methology may be extended to more than three variables, but some intensive computing issues need to be solved efficiently.

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