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On the Determination of Centers of Mass via Fractal Calculus and Its Applications in Board Games

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Abstract: This study introduces a novel approach to chess analysis based on center-of-mass dynamics and discrete fractal derivatives, offering an alternative framework for evaluating gameplay strategies. Unlike conventional methods that rely on exhaustive search and statistical simulations, our model provides a macroscopic perspective by analyzing the collective motion of pieces over time. By representing chess positions as a dynamic system in $\mathbb{R}^2$, we identify key movement patterns—such as oblique, parallel, and orthogonal trends—revealing strategic tendencies throughout the game. Additionally, fractal derivatives enable the detection of subtle momentum shifts and long-term imbalances, enhancing the understanding of decision-making processes. This approach is computationally efficient and adaptable, extending beyond chess to applications in sports analytics and real-time strategy games. These findings highlight the potential of interdisciplinary techniques in capturing complex strategic behavior within dynamic environments.

Keywords: chess dynamics; center of mass; fractal derivatives; symbolic mass assignment; nonlinear motion analysis; king trajectory; physics in chess

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1. Introduction

Chess is a rich subject of study for researchers interested in strategy, decision-making, and artificial intelligence. In this work, we proposed a novel approach to analyzing chess dynamics inspired by discrete physical models involving multiple particles. By assigning a specific mass to each chess piece based on its strategic importance, we tracked the center of mass of all pieces on the board to gain insights into the flow and distribution of power during the game. This method allowed us to study how the center of mass shifted over time and provided a macroscopic perspective on the game's progression.

Traditional chess analysis focuses on discrete board evaluations and heuristic-based algorithms [1], often overlooking the continuous evolution of strategic imbalances. How can we systematically quantify and track the macroscopic flow of a chess game to reveal long-term positional trends and critical transitions? To address this, we employ a methodology commonly used in physics to study particle systems, modeling chess dynamics through the weighted center of mass of the pieces, where mass is assigned based on strategic influence. This approach provides a macroscopic perspective on positional shifts, enabling the identification of long-term trends and key transitions in gameplay. By incorporating fractal derivatives, we refine this analysis to detect non-linear momentum variations that conventional differentiation methods fail to capture. The combination of these techniques allows for a deeper understanding of space control, strategic planning, and power distribution throughout a match, offering an alternative framework for both human analysis and AI-driven decision-making in chess.

The center of mass was determined by combining the assigned masses of the chess pieces with their discrete positions on the board, which were updated after every move. This calculation provided a macroscopic representation of the game's power distribution and allowed us to study how this balance shifted over time. To further analyze these dynamics, we computed the velocity and acceleration of the center of mass using fractal derivatives, as proposed in [2–4], with a discrete formulation tailored to fractal behaviors proposed in [5]. These derivatives were particularly effective for capturing non-linear changes, making them well-suited for identifying subtle transitions in gameplay.

Compared to conventional differentiation methods, the use of fractal derivatives significantly enhances the analysis of chess dynamics by capturing fine-grained variations in movement that standard discrete derivatives might overlook. Traditional differentiation assumes smooth and continuous functions, whereas chess movements are inherently discrete and often exhibit abrupt changes in momentum. Fractal derivatives accommodate these non-linear transitions, making them particularly useful in detecting shifts in strategic momentum, identifying critical turning points, and distinguishing between stable and volatile game phases.

Additionally, linear regressions were applied to the center of mass trajectories to uncover long-term trends, offering deeper insights into the flow of the game. The movements of the kings, as pivotal elements in the dynamics and objectives of chess, were also tracked and visualized.. This detailed analysis highlighted their critical influence on the progression and outcome of the match, further enriching our understanding of chess dynamics.

The assignment of mass values to chess pieces follows a rationale based on their strategic importance and mobility. Higher masses are given to pieces that exert greater control over the board and influence long-term game dynamics, such as queens and rooks, while lighter masses are assigned to pawns due to their limited movement and influence. The choice of mass distribution significantly affects the center of mass calculations; alternative assignments, such as equal mass for all pieces, would yield different trajectory patterns, potentially diminishing the ability to capture power imbalances accurately. By weighting the pieces according to their strategic impact, our model ensures that the center of mass dynamics reflect meaningful shifts in positional advantage.

In the context of velocity and acceleration computations, fractal derivatives offer a superior approach to traditional discrete derivatives by providing a more nuanced representation of movement tendencies. Conventional finite-difference methods may fail to capture intermittent fluctuations in piece dynamics, whereas fractal derivatives inherently account for irregular variations. This property enables a more refined detection of acceleration patterns, helping to reveal whether a player is gradually building positional pressure or suddenly shifting momentum.

A key parameter in our methodology is the choice of the fractional orders $\alpha = \beta = 1/2$ in the fractal derivative formulation. This value was selected heuristically as the central reference within the permissible range $\alpha, \beta \in (0, 1)$. Although this choice provides a balanced sensitivity to both short-term fluctuations and long-term trends, a formal sensitivity analysis will be conducted in future work to assess its impact on the interpretation of chess dynamics.

The main novelty of this study lies in applying physical modeling principles, particularly center of mass dynamics and fractal derivatives, to chess analysis. Unlike traditional methods that evaluate individual board positions or predict optimal moves, our approach provides a macroscopic perspective that captures the collective behavior of all pieces, revealing momentum shifts, long-term strategic imbalances, and critical transition points that may not be apparent through conventional analysis. By leveraging fractal derivatives—highly effective in detecting non-linear transitions—we uncover deep structural patterns in gameplay, enhancing our understanding of strategic evolution. This perspective enables the classification of playing styles, such as aggressive, positional, or dynamic, based on the center of mass movement and dispersion. Ultimately, our method offers a holistic view of gameplay dynamics, complementing traditional chess analysis and demonstrating the value of interdisciplinary approaches in understanding complex strategic decision-making.

Traditional chess engines such as Stockfish and AlphaZero used well-established methods to evaluate positions and determine optimal moves. Stockfish employed deterministic algorithms, combining handcrafted evaluation functions with alpha-beta pruning to explore move trees efficiently, while AlphaZero leverages deep reinforcement learning and Monte Carlo Tree Search (MCTS) to autonomously discover strategies by simulating countless games [6]. Although powerful, these methods primarily focused on optimizing specific moves and evaluating isolated board states rather than analyzing global patterns in gameplay

Monte Carlo Tree Search, Carlo Tree Search, particularly when integrated with neural networks, was widely used to improve decision-making in chess engines [7]. This method evaluates potential moves by simulating numerous game outcomes, providing computational efficiency and depth in search trees. However, like traditional engines, it emphasizes localized optimization rather than a macroscopic understanding of the game's dynamics.

Recent advancements in machine learning have introduced novel techniques for chess analysis. For example, in [8], a model was developed to estimate player ratings directly from game moves and clock times. This architecture combined convolutional neural networks (CNNs) to learn positional features with bidirectional long short-term memory networks (BiLSTMs) to process temporal data, achieving a mean absolute error of 182 rating points. Similarly, ref. [9] proposed a model that predicted player ratings based on move sequences and metadata, emphasizing its potential for enhancing rating systems and detecting irregularities such as cheating.

Another innovative approach to chess analytics involves clustering techniques to study the impact of opening strategies. In [10], the authors applied the DBSCAN clustering algorithm to categorize opening patterns and assess their influence on game outcomes. By analyzing player ratings and one-hot encoded move sequences, the study highlighted the strategic implications of openings and their role in shaping the course of a match.

Although these approaches excelled in move prediction, position evaluation, and specific aspects of gameplay analysis, they often lacked a comprehensive view of the game's overall dynamic. Our model offers a unique alternative by focusing on the collective behavior of all pieces on the board. By leveraging the concept of the center of mass and incorporating fractal derivatives to analyze non-linear trajectories, we provided a robust framework for understanding the power distribution and its evolution throughout the game. Inspired by physical systems where collective properties are used to analyze complex interactions, this approach identifies critical moments and patterns in gameplay that may otherwise remain hidden. This macroscopic perspective complements existing methods and provides a new lens through which the intricate dynamics of chess can be studied.

To implement this model, we employed computational tools using Python (3.13.2) [11], combining fractal derivatives and center of mass analysis to predict the dynamics of chess gamescombining fractal derivatives and center of mass analysis to predict the dynamics of chess games. This computational approach enables the classification of gameplay into categories such as offensive, defensive, central, or flank-oriented strategies. By analyzing matches from grandmasters like Kasparov, Karpov, Carlsen, Nepomniachtchi, Fischer, and Spassky, we aim to reveal distinct strategic patterns and behaviors. Python's versatility allowed for robust data processing and visualization, ensuring a detailed and systematic study of the gameplay dynamics. This approach highlights the potential of combining computational tools with physical principles to deepen our understanding of chess.

The structure of this paper is as follows: Section 2 describes the methodology employed in this study. Section 2.1 introduces the fundamental principles of fractal calculus and the discretization approach used. Section 2.2 explains the computation of the center of mass, while Section 2.4 presents illustrative examples. Finally, Section 3 provides the conclusions of this work.

2. Methods

In this section, we introduce the mathematical and physical concepts employed to model the dynamics of chess gameplay. The framework combines principles inspired by classical mechanics, such as center of mass calculations and kinematics, with advanced tools like fractal and discrete fractal derivatives. These methodologies provide a robust means to analyze non-linear and complex interactions between chess pieces over time. Additionally, the specific mass assignments for each piece and their corresponding positional mapping on the chessboard are described, laying the foundation for a macroscopic representation of the game's dynamics. By integrating these techniques, we aim to capture both the immediate and long-term strategic patterns inherent in chess gameplay.

The steps to follow in our proposed methodology are as follows:

- Treat each chess piece as a particle with a variable mass, where the mass reflects the piece's strategic significance in the game.
- Track the center of mass (CoM) of all active pieces rather than focusing on the individual movements of each piece. This provides a macroscopic perspective of the game's dynamics.
- Use discrete fractal derivatives to analyze the velocity of the CoM, capturing non-linear dynamics and changes at multiple scales. This method enables the identification of subtle shifts in velocity and strategic trends over time.
- Apply this framework to study the long-term strategic evolution of the game, allowing us to uncover patterns of positional advantages, velocity shifts, and decision-making tendencies.

2.1. Fractal and Discrete Fractal Derivatives

The fractal derivative is a generalization of the classical derivative, designed to describe dynamics exhibiting self-similarity and non-linear scaling. As defined in [2–4,12,13], the fractal derivative of a function $f(t)$ is given by:

$$\frac{df(t)}{dt^{\alpha}} = \lim_{s \to t} \frac{f(t) - f(s)}{|t|^{\alpha} - |s|^{\alpha}}, \tag{1}$$

where $\alpha \in (0, 1]$ by definition is the fractal order. This formulation allows for the modeling of processes with non-integer differentiation scales.

In addition to the standard fractal derivative, the two-scale fractal derivative offers an alternative approach for capturing multi-scale dynamics in chess gameplay. This method is particularly useful for analyzing systems where behavior varies across different scales [14], such as the interplay between short-term tactical moves and long-term strategic planning. Future work could explore the application of two-scale fractal derivatives to enhance the model's ability to capture complex, multi-scale interactions in chess.

In a more generalized form, we can consider the fractal derivative (1) as follows:

$$\frac{d^\beta f(t)}{dt^\alpha} = \lim_{s \to t} \frac{f^\beta(t) - f^\beta(s)}{|t|^\alpha - |s|^\alpha}, \tag{2}$$

where $\beta \in (0, 1]$ by definition. In discrete systems, the concept is extended to a discrete fractal derivative. Following the definition provided in [5], the discrete fractal derivative for a sequence f_n is formulated as:

$$\begin{aligned} \frac{\Delta^\beta f(t_n)}{\Delta t^\alpha} &= \frac{\beta f^{\beta-1}(t_n)(f(t_{n+1}) - f(t_n))}{\alpha t_n^\alpha (t_{n+1} - t_n)} \\ &= \frac{\beta f_n^{\beta-1}(f_{n+1} - f_n)}{\alpha h (nh)^\alpha} \\ &= \frac{\beta}{\alpha} \cdot \frac{f_n^{\beta-1}}{(nh)^\alpha} \cdot \frac{\Delta f_n}{h}, \end{aligned} \tag{3}$$

where α modifies the scaling of the discrete increments, enabling sensitivity to discrete non-linear behaviors in a fractal context. In our specific case, like $h = 1$ we obtain:

$$\frac{\Delta^\beta f(t_n)}{\Delta t^\alpha} = \frac{\beta}{\alpha} \cdot \frac{f_n^{\beta-1}}{(n)^\alpha} \cdot \Delta f_n \tag{4}$$

Although the current study employs a fixed fractal order $\alpha = \beta = 0.5$, future work could explore the use of He-Liu [15] fractal dimension formulas to dynamically calculate the fractal order during gameplay. This approach would allow for a more adaptive analysis of chess dynamics, capturing changes in strategic complexity as the game progresses.

2.2. Center of Mass Calculation

To analyze the dynamics of a chess game, the center of mass is calculated by assigning specific masses to each chess piece. These masses are determined based on the strategic importance and movement capabilities of each piece. The assigned values are outlined in Table 1, along with their classical symbols used in chess notation.

Table 1. Masses assigned to chess pieces based on their strategic influence and position on the board.

Piece	Symbol	Mass (Points)
Pawn (Initial)	P	1.0
Pawn (1 square advanced)	P	1.5
Pawn (2 squares advanced)	P	2.0
Pawn (3 squares advanced)	P	3.0
Pawn (4 squares advanced)	P	4.0
Pawn (5 squares advanced)	P	5.0
Pawn (Promoted)	P	9.0
Knight	N	3.0
Bishop	B	3.0
Rook	R	5.0
Queen	Q	9.0
King	K	3.5

Mass values reflect the increasing importance of pawns as they advance and the relative strategic weight of major pieces.

Remark 1. *The value of a pawn evolves as it advances across the board, reflecting its increasing potential and influence in the game:*

- ***Row 2:*** *The pawn begins with a base value of 1 point, representing its status as the least valuable piece in the game.*
- ***Row 3:*** *As the pawn progresses, its value rises slightly to 1.5 points, reflecting its greater potential to participate in the game.*
- ***Row 4:*** *At this stage, the pawn gains strategic importance by controlling more space, increasing its value to approximately 2 points.*
- ***Row 5:*** *Here, the pawn becomes a more aggressive and integral part of the game's structure, with its value climbing to 3 points.*
- ***Row 6:*** *Approaching promotion, the pawn becomes even more critical, with its value rising to around 4 points.*
- ***Row 7:*** *In the penultimate row, the pawn's value peaks at 5 points, signifying its proximity to becoming a queen.*
- ***Row 8:*** *Upon reaching the eighth row and promoting, the pawn transforms into a queen, attaining the maximum value of 9 points.*

The assignment of symbolic mass values to chess pieces provides a macroscopic view of power distribution on the board. In future work, the use of two-scale fractal dimensions could offer a more nuanced understanding of how this distribution evolves over time, capturing both local and global strategic shifts. This approach would allow for a more detailed analysis of the interplay between short-term tactical moves and long-term strategic planning.

The chessboard is modeled as a Cartesian coordinate system $[0,8] \times [0,8]$, where each square is represented by its center point in this space. For a given piece positioned at (x_i, y_i) with a mass m_i, the center of mass of all N pieces on the board is calculated as follows:

$$x_{\text{cm}}(t) = \frac{\sum_{i=1}^{N} m_i(t) x_i(t)}{\sum_{i=1}^{N} m_i(t)}, \quad y_{\text{cm}}(t) = \frac{\sum_{i=1}^{N} m_i(t) y_i(t)}{\sum_{i=1}^{N} m_i(t)}. \tag{5}$$

This formulation provides a macroscopic representation of the distribution of power across the board. By monitoring how the center of mass evolves over time, we gain insights into strategic transitions and shifts in positional balance.

The mass assignments also highlight the nuanced roles of the pieces. For example, the incremental increase in pawn mass as it advances reflects its growing influence as it approaches promotion. Similarly, the relatively high mass of the queen underscores its dominant strategic importance, while the king's assigned value balances its pivotal yet defensive role in the game. This abstraction not only facilitates physical modeling but also aligns with intuitive strategic considerations in chess.

By incorporating these values, our approach emphasizes both the individual contributions of pieces and their collective dynamics, paving the way for a detailed macroscopic analysis of chess gameplay.

2.3. Velocity and Acceleration in Classical and Fractal Derivatives

In classical mechanics, the velocity of a position vector $\mathbf{r}(t) = (x(t), y(t))$ is defined as the first derivative with respect to time:

$$\mathbf{v}(t) = \frac{d\mathbf{r}(t)}{dt} = \left(\frac{dx(t)}{dt}, \frac{dy(t)}{dt} \right), \tag{6}$$

The acceleration, which measures the rate of change of velocity, is given by the second derivative of the position vector:

$$\mathbf{a}(t) = \frac{d^2\mathbf{r}(t)}{dt^2} = \left(\frac{d^2x(t)}{dt^2}, \frac{d^2y(t)}{dt^2}\right), \tag{7}$$

To analyze the complex and non-linear dynamics in chess gameplay, fractal derivatives provide a generalized framework for defining velocity and acceleration. These derivatives, as introduced in [2], are denoted by $\mathbf{v}^{\beta,\alpha}(t)$and $\mathbf{a}^{\beta,\alpha}(t)$, where (β, α) represents the order of the derivative.

Given the discrete nature of chess gameplay, it is necessary to reformulate these fractal derivatives in a discrete framework. The discrete fractal derivative, detailed in [5], and expressed in Equation (4), is defined using the fractal orders β and α. Here, f_n represents the function's value at step n, and $\frac{\Delta^\beta f(t_n)}{\Delta t^\alpha}$ denotes the discrete fractal derivative.

Using this discrete formulation, the fractal velocity and acceleration are computed as follows:

$$\mathbf{v}^{\beta,\alpha}(t_n) = \frac{\Delta^\beta \mathbf{r}(t_n)}{\Delta t^\alpha} = \left(\frac{\Delta^\beta x(t_n)}{\Delta^\alpha t}, \frac{\Delta^\beta y(t_n)}{\Delta^\alpha t}\right) = \frac{\beta}{\alpha n^\alpha}\left(x_n^{\beta-1}\Delta x_n, y_n^{\beta-1}\Delta y_n\right), \tag{8}$$

$$\mathbf{a}^{\beta,\alpha}(t_n) = \frac{\Delta^\beta \mathbf{v}(t_n)}{\Delta t^\alpha} = \left(\frac{\Delta^\beta v_x(t_n)}{\Delta^\alpha t}, \frac{\Delta^\beta v_y(t_n)}{\Delta^\alpha t}\right) = \frac{\beta}{\alpha n^\alpha}\left(v_{x_n}^{\beta-1}\Delta v_{x_n}, v_{y_n}^{\beta-1}\Delta v_{y_n}\right), \tag{9}$$

This framework, combining fractal and discrete calculus, is particularly suited to capture the intricate and non-linear shifts in the center of mass during chess gameplay. By modeling velocity, acceleration, and jerk in this way, the analysis can reveal subtle transitions and patterns in the dynamics of the game, offering a richer perspective on gameplay progression and strategic flow.

2.4. Illustrative Examples of Gameplay Analysis

This section presents detailed examples illustrating how the proposed mathematical framework can be applied to analyze chess gameplay. By examining notable matches, we demonstrate shifts in the center of mass, velocity, acceleration, and other dynamic properties. These cases emphasize the framework's practical utility in uncovering strategic and non-linear patterns, offering fresh perspectives on chess analysis.

We can plot the velocity $\mathbf{v}^{\beta,\alpha}(t_n)$ and acceleration $\mathbf{a}^{\beta,\alpha}(t_n)$ as defined in (8) and (9). For this study, we consider $\alpha = \beta = 1/2$, which corresponds to the classical case of fractional derivatives. Under this assumption, the fractal fractional discrete velocity and fractal fractional discrete acceleration are expressed as:

$$\mathbf{v}^{1/2,1/2}(t_n) = \frac{1}{\sqrt{n}}\left(\frac{\Delta x_n}{\sqrt{x_n}}, \frac{\Delta y_n}{\sqrt{y_n}}\right), \tag{10}$$

$$\mathbf{a}^{1/2,1/2}(t_n) = \frac{1}{\sqrt{n}}\left(\frac{\Delta v_{x_n}}{\sqrt{v_{x_n}}}, \frac{\Delta v_{y_n}}{\sqrt{v_{y_n}}}\right). \tag{11}$$

2.4.1. Case Study 1: Magnus Carlsen vs. Ian Nepomniachtchi

Magnus Carlsen, regarded as one of the greatest chess players in history, faced Ian Nepomniachtchi in the World Chess Championship match on 3 December 2021. This game is widely celebrated for its exceptional display of strategic depth and tactical precision. Figure 1 visualizes the movement patterns of the pieces during this match. The analysis reveals a generally symmetric distribution of movements, with white pieces exhibiting greater mobility. Black pieces, on the other hand, are predominantly concentrated near the center of the x-coordinates, reflecting a more defensive posture.

In this exciting match, Magnus Carlsen (White) faced Ian Nepomniachtchi (Black) in a thrilling contest of strategic depth. The game unfolded as follows: 1. d4 Nf6 2. Nf3 d5 3. g3 e6 4. Bg2 Be7 5. O-O O-O 6. b3 c5 7. dxc5 Bxc5 8. c4 dxc4 9. Qc2 Qe7 10. Nbd2 Nc6 11. Nxc4 b5 12. Nce5 Nb4 13. Qb2 Bb7 14. a3 Nc6 15. Nd3 Bb6 16. Bg5 Rfd8 17. Bxf6 gxf6 18. Rac1 Nd4 19. Nxd4 Bxd4 20. Qa2 Bxg2 21. Kxg2 Qb7+ 22. Kg1 Qe4 23. Qc2 a5 24. Rfd1 Kg7 25. Rd2 Rac8 26. Qxc8 Rxc8 27. Rxc8 Qd5 28. b4 a4 29. e3 Be5 30. h4 h5 31. Kh2 Bb2 32. Rc5 Qd6 33. Rd1 Bxa3 34. Rxb5 Qd7 35. Rc5 e5 36. Rc2 Qd5 37. Rdd2 Qb3 38. Ra2 e4 39. Nc5 Qxb4 40. Nxe4 Qb3 41. Rac2 Bf8 42. Nc5 Qb5 43. Nd3 a3 44. Nf4 Qa5 45. Ra2 Bb4 46. Rd3 Kh6 47. Rd1 Qa4 48. Rda1 Bd6 49. Kg1 Qb3 50. Ne2 Qd3 51. Nd4 Kh7 52. Kh2 QE4 53. Rxa3 Qxh4+ 54. Kg1 QE4 55. Ra4 Be5 56. Ne2 Qc2 57. R1a2 Qb3 58. Kg2 Qd5+ 59. f3 Qd1 60. f4 Bc7 61. Kf2 Bb6 62. Ra1 Qb3 63. Re4 Kg7 64. Re8 f5 65. Raa8 Qb4 66. Rc8 Ba5 67. Rc1 Bb6 68. Re5 Qb3 69. Re8 Qd5 70. Rcc8 Qh1 71. Rc1 Qd5 72. Rb1 Ba7 73. Re7 Bc5 74. Re5 Qd3 75. Rb7 Qc2 76. Rb5 Ba7 77. Ra5 Bb6 78. Rab5 Ba7 79. Rxf5 Qd3 80. Rxf7+ Kxf7 81. Rb7+ Kg6 82. Rxa7 Qd5 83. Ra6+ Kh7 84. Ra1 Kg6 85. Nd4 Qb7 86. Ra2 Qh1 87. Ra6+ Kf7 88. Nf3 Qb1 89. Rd6 Kg7 90. Rd5 Qa2+ 91. Rd2 Qb1 92. Re2 Qb6 93. Rc2 Qb1 94. Nd4 Qh1 95. Rc7+ Kf6 96. Rc6+ Kf7 97. Nf3 Qb1 98. Ng5+ Kg7 99. Ne6+ Kf7 100. Nd4 Qh1 101. Rc7+ Kf6 102. Nf3 Qb1 103. Rd7 Qb2+ 104. Rd2 Qb1 105. Ng1 Qb4 106. Rd1 Qb3 107. Rd6+ Kg7 108. Rd4 Qb2+ 109. Ne2 Qb1 110. e4 Qh1 111. Rd7+ Kg1 112. Rd4 Qh2+ 113. Ke3 h4 114. gxh4 Qh3+ 115. Kd2 Qxh4 116. Rd3 Kf8 117. Rf3 Qd8+ 118. Ke3 Qa5 119. Kf2 Qa7+ 120. Re3 Qd7 121. Ng3 Qd2+ 122. Kf3 Qd1+ 123. Re2 Qb3+ 124. Kg2 Qb7 125. Rd2 Qb3 126. Rd5 Ke7 127. Re5+ Kf7 128. Rf5+ Ke8 129. e5 Qa2+ 130. Kh3 Qe6 131. Kh4 Qh6+ 132. Nh5 Qh7 133. e6 Qg6 134. Rf7 Kd8 135. f5 Qg1 136. Ng7 1-0.

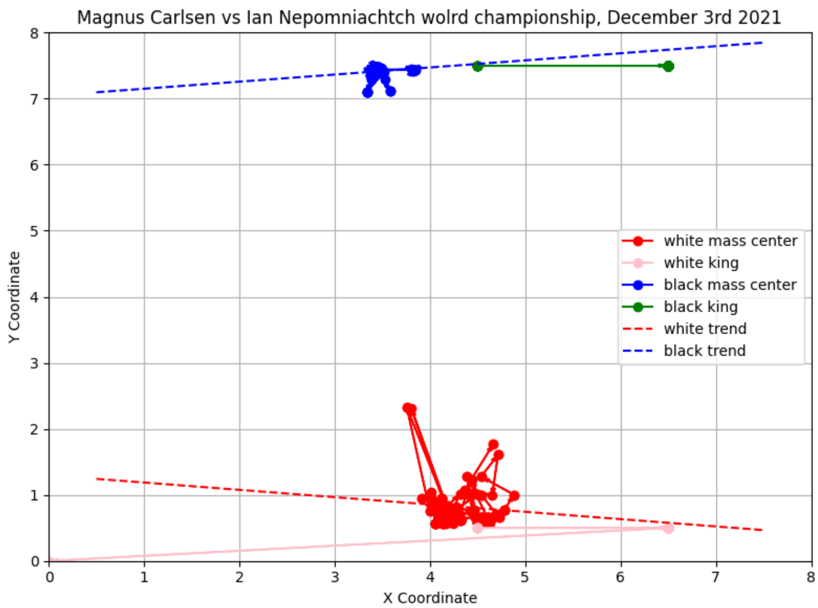

Figure 1. Piece movement trends in the Carlsen vs. Nepomniachtchi match, World Chess Championship, 3 December 2021. It can be observed that the lines tend to follow an oblique pattern with a low slope, indicating that both players had attacking intentions on the left flank.

We can plot the velocity $\mathbf{v}^{1/2,1/2}(t_n)$ using the fractal derivatives with $\alpha = \beta = 0.5$, as shown in Figure 2. In this analysis, the graph visualizes the dynamic shifts of the pieces in the game, revealing distinct patterns of movement. The velocities in the plot show opposite directions for the two players, which is expected since, while the black player is attacking the white side, the white player is simultaneously attacking the black side.

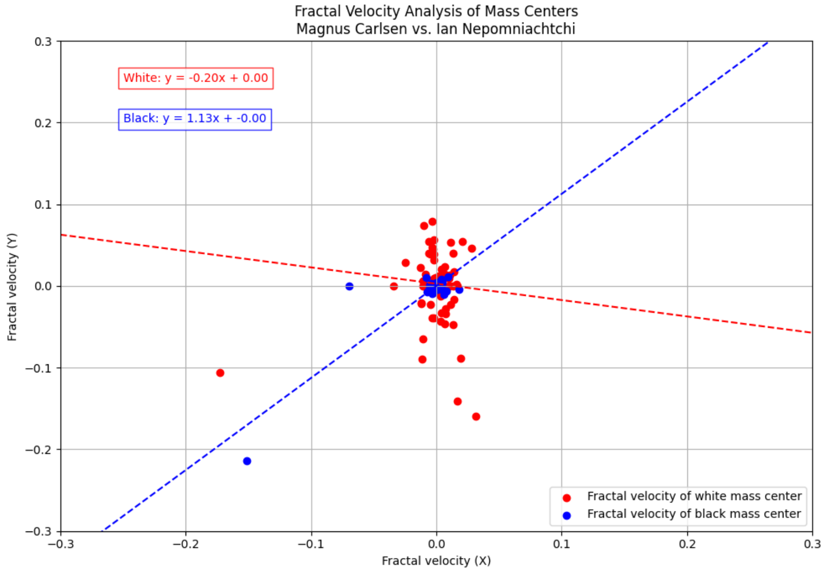

Figure 2. Plot of velocity using fractal derivatives with $\alpha = \beta = 0.5$. The opposite directions of velocity trends reflect the attacking strategies of both players, with the black player advancing toward the white side and the white player attacking the black side.

Given the significance of the Magnus Carlsen vs. Ian Nepomniachtchi match—one of the longest and most strategically complex encounters in chess history—it is pertinent to explore the model's behavior under different fractal orders. Specifically, employing $\alpha = 0.4$ and $\beta = 0.5$, as depicted in Figure 3, reveals a notable transformation in the angular relationships between trajectory lines, resulting in a more perpendicular orientation. This observation suggests a potential sensitivity of the model to the choice of fractal parameters. A comprehensive sensitivity analysis of α and β will be conducted in future work to systematically assess their influence on the interpretation of gameplay dynamics.

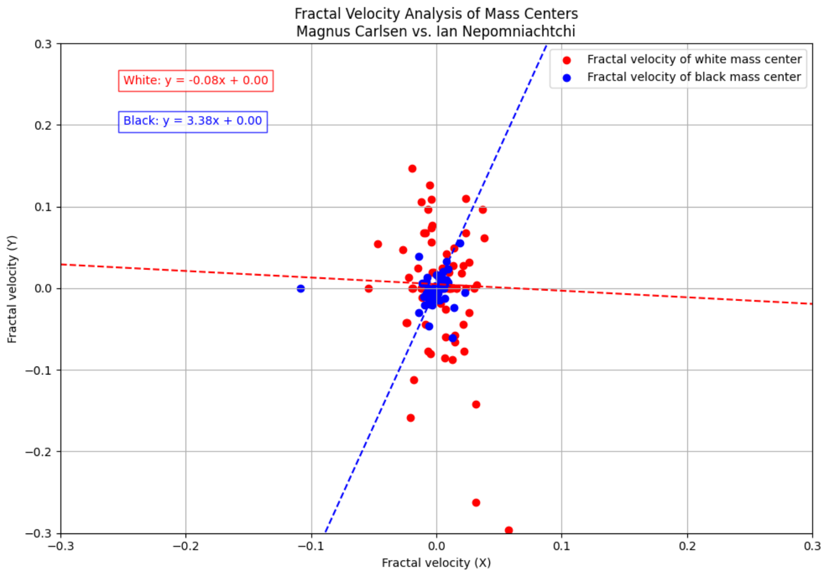

Figure 3. Velocity plot using fractal derivatives with $\alpha = 0.4$ and $\beta = 0.5$. As opposing directional trends remain evident, the angular relationships between vectors become more pronounced, exhibiting a greater degree of perpendicularity.

The displacement trajectories exhibit a symmetrical divergence from the respective castling sides, while the computed fractal velocities display an almost perpendicular orientation. This phenomenon indicates that, although piece movements were predominantly directed toward the left flank, velocity vectors were oriented toward the opponent's position, thereby highlighting a distinct tendency toward offensive play.

2.4.2. Case Study 2: Kasparov—Karpov World Championship Match (1985), Moscow URS, Rd 16, Oct-15

Garry Kasparov and Anatoly Karpov, two of the most legendary chess players in history, faced each other in a critical 16th-round game during the 1985 World Chess Championship in Moscow. This game is renowned for its intricate strategic depth and the distinct dynamics of piece movements. Notably, the analysis reveals a parallel alignment in the distribution of their pieces, with significant weight concentrated on the queenside of the black pieces, showcasing a strong positional focus from both players.

The game unfolded as follows: 1. e4 c5 2. Nf3 e6 3. d4 cxd4 4. Nxd4 Nc6 5. Nb5 d6 6. c4 Nf6 7. Nc3 a6 8. Na3 d5 9. cxd5 exd5 10. exd5 Nb4 11. Be2 Bc5 12. O-O O-O 13. Bf3 Bf5 14. Bg5 Re8 15. Qd2 b5 16. Rad1 Nd3 17. Nab1 h6 18. Bh4 b4 19. Na4 Bd6 20. Bg3 Rc8 21. b3 g5 22. Bxd6 Qxd6 23. g3 Nd7 24. Bg2 Qf6 25. a3 a5 26. axb4 axb4 27. Qa2 Bg6 28. d6 g4 29. Qd2 Kg7 30. f3 Qxd6 31. fxg4 Qd4+ 32. Kh1 Nf6 33. Rf4 Ne4 34. Qxd3 Nf2+ 35. Rxf2 Bxd3 36. Rfd2 Qe3 37. Rxd3 Rc1+ 38. Nb2 Qf2 39. Nd2 Rxd1+ 40. Nxd1 Re1+ 0-1.

As illustrated in Figure 4, the displacement graphs demonstrate a pronounced parallel alignment in the movement tendencies of the pieces. This alignment is particularly evident on the queenside of the black pieces, where Karpov concentrated his efforts to establish a defensive stronghold. This strategic emphasis on the queenside reflects Karpov's methodical approach to countering Kasparov's aggressive tactics.

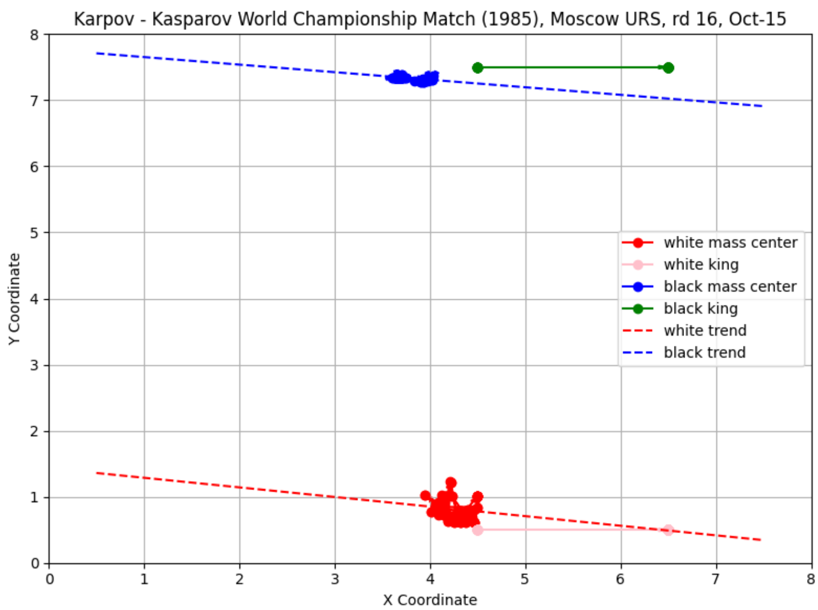

Figure 4. Piece movement trends in the Kasparov vs. Karpov match, World Chess Championship, 15 October 1985.

The velocity analysis, as shown in Figure 5, computed using fractal derivatives with $\alpha = \beta = 0.5$, further reinforces the observation of parallel tendencies in the movement patterns of the pieces. This analysis highlights the players' strategic efforts to stabilize their respective positions while maintaining pressure on the queenside. Such dynamics underline the profound positional understanding exhibited by both Kasparov and Karpov during this iconic encounter, solidifying its place as one of the most celebrated games in chess history.

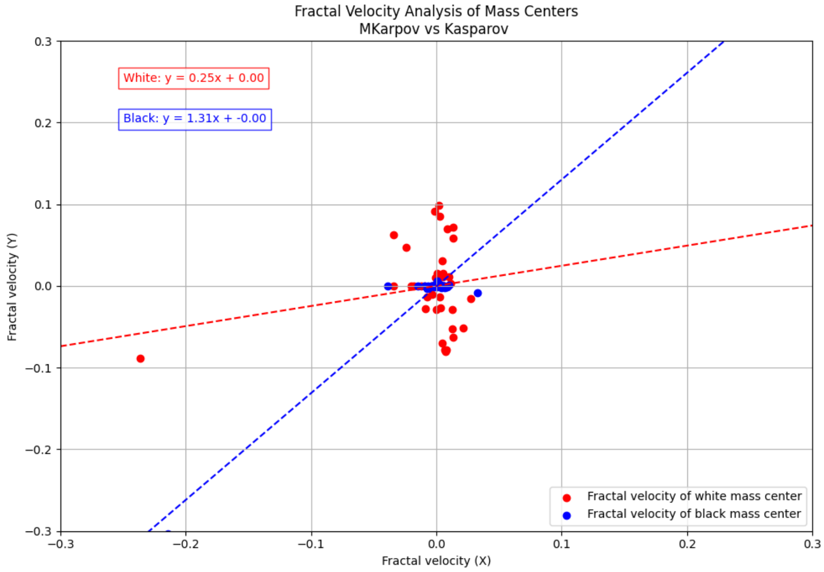

Figure 5. Velocity analysis using fractal derivatives with $\alpha = \beta = 0.5$. The parallel trends emphasize the strategic queenside focus by both players.

2.4.3. Case Study 3: Spassky—Fischer World Championship Match (1972), Reykjavik ISL, Rd 3, Jul-16

The 1972 World Chess Championship match between Boris Spassky and Bobby Fischer, held in Reykjavik, Iceland, is often regarded as one of the greatest and most historically significant encounters in chess history. This match, dubbed the "Match of the Century", not only marked a pivotal moment in the Cold War rivalry between the United States and the Soviet Union but also showcased Fischer's unparalleled genius on the board. The third round of this iconic event, analyzed here, featured the Benoni Defense: Knight's Tour Variation (A61), highlighting Fischer's creativity and dynamic play.

The game proceeded as follows: 1. d4 Nf6 2. c4 e6 3. Nf3 c5 4. d5 exd5 5. cxd5 d6 6. Nc3 g6 7. Nd2 Nbd7 8. e4 Bg7 9. Be2 O-O 10. O-O Re8 11. Qc2 Nh5 12. Bxh5 gxh5 13. Nc4 Ne5 14. Ne3 Qh4 15. Ad2 Ng4 16. Nxg4 hxg4 17. Af4 Qf6 18. g3 Ad7 19. a4 b6 20. Tfe1 a6 21. Re2 b5 22. Rae1 Qg6 23. b3 Re7 24. Qd3 Rb8 25. axb5 axb5 26. b4 c4 27. Qd2 Rbe8 28. Re3 h5 29. R3e2 Kh7 30. Re3 Kg8 31. R3e2 Axc3 32. Dxc3 Rxe4 33. Txe4 Rxe4 34. Txe4 Dxe4 35. Ah6 Dg6 36. Ac1 Qb1 37. Rf1 Af5 38. Re2 De4+ 39. De3 Qc2+ 40. Dd2 Qb3 41. Dd4 Ad3+ 0-1.

As illustrated in Figure 6, the movement dynamics reveal parallel tendencies in piece distribution, with a slight bias toward the kingside. The white pieces exhibited greater activity, reflecting Fischer's dominant control of the board. This superiority is evident in the computed center of mass, which indicates a broader spatial influence for Fischer's pieces compared to Spassky's.

The velocity analysis, depicted in Figure 7, highlights the central role of the a1–h8 diagonal in the game. Although the trend lines are not strictly perpendicular, the opposing directional tendencies of the players' movements are clearly visible. This dynamic illustrates Fischer's ability to leverage the long diagonal to apply pressure, ultimately forcing Spassky into a defensive posture. The interplay of piece movement and velocity underscores the strategic depth and brilliance of this historic game.

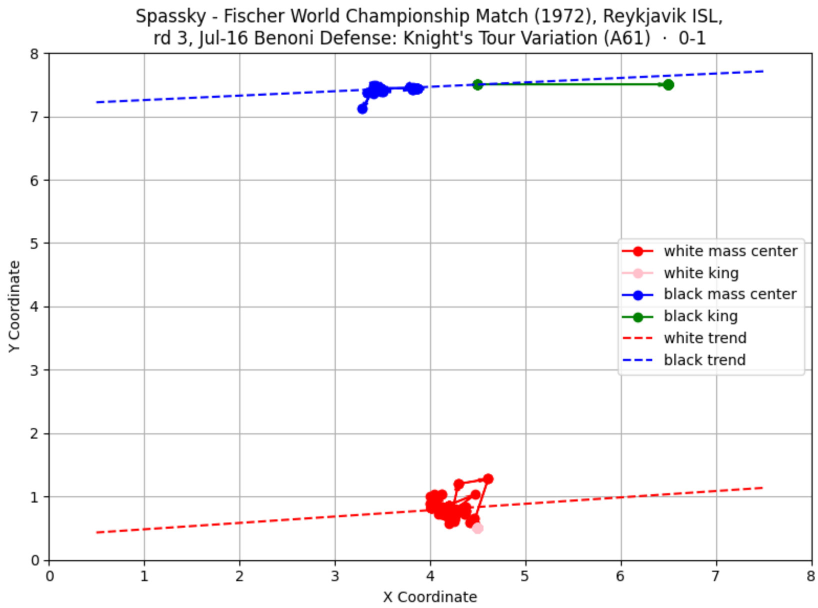

Figure 6. Piece movement trends in the Spassky vs. Fischer match, World Chess Championship, 16 July 1972.

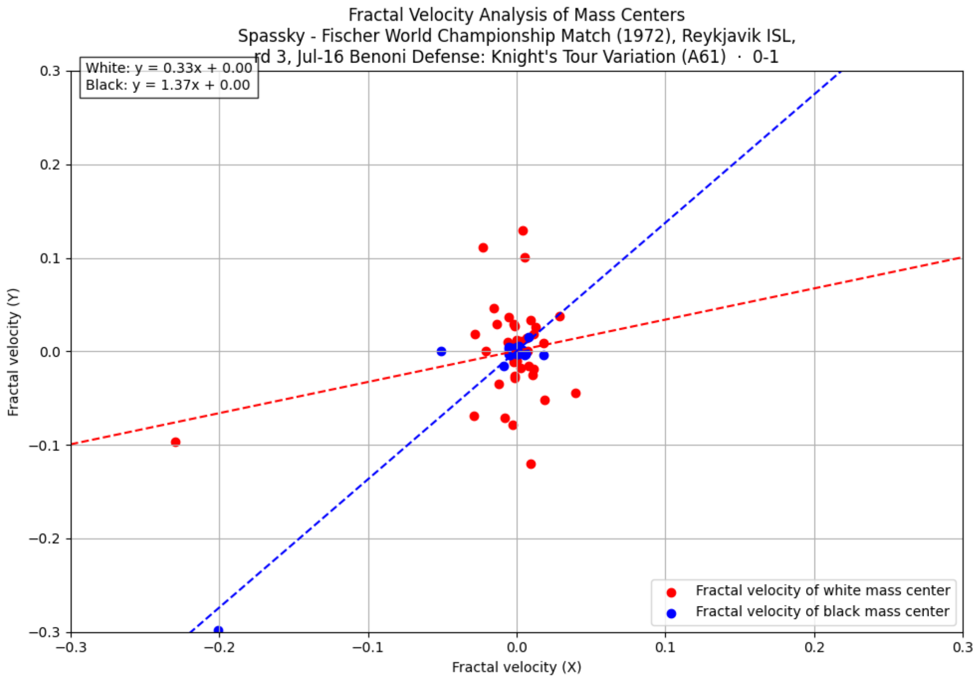

Figure 7. Velocity analysis using fractal derivatives with $\alpha = \beta = 0.5$. The trends emphasize the focus on the a1-h8 diagonal and opposing directional tendencies. The red and blue lines represent the same as in Figure 5.

3. Discussion

The utilization of a method based on centers of mass provides an effective framework for analyzing the dynamics of a group of elements without the necessity of studying each individual component in isolation. This approach enables us to describe the general trajectories of piece movements and to uncover patterns exhibited throughout the game. By aggregating the collective behavior of the pieces into a simplified representation, valuable insights can be derived regarding the strategies and tendencies of the players.

In the context of chess, the trends in movement trajectories, visualized in an $\mathbb{R}^2$ space, can exhibit three primary orientations:

- **Oblique Lines:** These indicate that both players are directing their forces toward the same regions of the board, albeit in opposite directions. Such trends, as observed in the Carlsen vs. Nepomniachtchi match (Section 2.4.1), reflect a "crossed style" of play, where the strategic focus creates tension along intersecting paths. In extreme cases, this dynamic can lead to orthogonal tendencies.
- **Parallel Lines:** When movement trends are parallel, as seen in the Kasparov vs. Karpov (Section 2.4.2) and Spassky vs. Fischer (Section 2.4.3) matches, the players' strategies align more closely. Although the directions are nearly identical, the movements often occur in opposite senses, showcasing a diagonal pattern that emphasizes positional control and strategic balance.
- **Orthogonal Lines:** This pattern, while less common, signifies that players are focusing their strategies in entirely distinct regions of the board, minimizing direct confrontation while attempting to create opportunities through positional play.

In addition to displacement analysis, the study of velocities provides critical insights into the principal tendencies of piece movements and the directionality of players' strategies. By analyzing the velocity vectors, one can observe the primary focus of the pieces, akin to the physical velocity of a body composed of many particles in a discrete system or a continuous medium in a fluid dynamics context. This analogy highlights the interconnected nature of discrete movements in chess and continuous dynamics in physical systems.

The combined understanding of displacement and velocity trends enables the development of predictive methods and the identification of behavioral patterns exhibited by players. Such insights can be instrumental in training chess engines or game-playing artificial intelligence (AI) systems. By recognizing the tendencies and preferred strategies of specific players, these systems can adapt and optimize their play to counter or emulate such styles effectively.

Although chess is fundamentally a game of discrete moves, the analysis of its dynamics can extend to systems that exhibit continuous or multi-scale behavior. For example, the principles explored here can be applied to more complex scenarios, such as the movement dynamics in soccer matches, RPG games like *Age of Empires*, or the modeling of unit platoons in real-time strategy games. The continuity and scalability of these systems provide fertile ground for exploring analogous behavior and broadening the application of the methodologies developed. However, further studies are needed to validate the method's effectiveness in these contexts, particularly in adapting the fractal parameters and mass assignments to capture the unique dynamics of each system. This would ensure that the method remains robust and versatile across diverse strategic environments.

The advantage of employing a center-of-mass method lies in its ability to reduce the complexity of analysis. Instead of focusing on multiple individual components, the behavior of the system can be summarized through a small number of centers of mass, offering a clear and computationally efficient way to identify trends and overall dynamics.

The implementation of fractional derivatives enhances this approach by introducing the order of differentiation as an adjustable parameter, enabling a finer calibration of the method to suit specific scenarios and providing a more nuanced analysis of the system's behavior. Fractional calculus extends the range of tools available for studying two-dimensional phenomena, bridging classical physics with non-conventional mathematics. Furthermore, the application of two-scale fractal dimensions could significantly improve the predictive accuracy of the method by capturing multi-scale dynamics in chess gameplay. Future studies should investigate how these dimensions vary during different phases of the game and their impact on the overall power distribution. This approach would allow for a deeper analysis of the interplay between short-term tactical moves and long-term strategic planning, offering richer insights into the evolution of gameplay dynamics.

Moreover, the fractal dimensions or the order of the fractal derivative may change during the game, reflecting shifts in strategic complexity and momentum. Future work should investigate the rules governing these changes, as they could reveal critical transitions and patterns in players' strategies. For example, a sudden increase in fractal dimension might indicate a shift from positional play to tactical aggression, while a decrease could signal a consolidation of defensive structures. Understanding these variations would provide a richer perspective on the dynamics of chess and other strategic games.

4. Conclusions

The proposed method demonstrates significant potential for analyzing chess dynamics. However, its applicability to other board games, such as Go (Weiqi)—one of the most intricate and historically significant strategy games—requires further validation and refinement. Future research should investigate how this framework can be adapted to capture the complexities of various board games and real-world scenarios, including team sports and real-time strategy games [16]. This would involve optimizing mass assignments, adjusting fractal parameters, and refining analytical models to ensure their robustness and accuracy across diverse contexts. Expanding this methodology could provide new insights into strategic decision-making beyond chess, enhancing our understanding of competitive and cooperative dynamics in different domains.

This methodology has the potential to be a powerful tool for understanding general trends in various games and other dynamic systems. For instance, by employing this algorithm, one can measure the dispersion of units, evaluate control over pieces, and quantify strategic advantages through the "weight" of total units. By doing so, it becomes possible to assess which player or team holds a positional or strategic edge, providing a quantitative measure of dominance in the game.

Compared to Monte Carlo Tree Search (MCTS), which leverages statistical sampling and extensive simulations to evaluate move sequences, our approach based on discrete fractal derivatives provides a structured and macroscopic analysis of chess dynamics. Although MCTS excels in tactical depth, discovering optimal moves through self-play and probabilistic exploration, it lacks a direct mechanism to analyze global patterns in gameplay. In contrast, our method captures long-term strategic imbalances and critical transitions by analyzing the temporal evolution of the center of mass. Additionally, it is computationally efficient since it does not require thousands of simulations but instead extracts insights from the physical modeling of power distribution.

However, our approach also has limitations compared to MCTS. Since fractal derivatives focus on macroscopic trends rather than specific move evaluations, they do not provide direct recommendations for optimal play. MCTS, by contrast, can generate precise move suggestions and adapt dynamically to various game scenarios. Furthermore, the effectiveness of our method depends on the chosen mass assignments and parameter tuning, which may introduce heuristic biases. MCTS benefits from reinforcement learning to refine its decision-making, whereas our method requires predefined modeling assumptions that may not generalize to all playing styles. Despite these differences, both approaches offer complementary insights: MCTS is highly effective in tactical decision-making, while fractal analysis provides a broader understanding of strategic evolution.

The proposed methodology, grounded in the well-established physical concept of center of mass and enhanced by velocity analysis and fractional calculus, provides a structured framework for quantifying strategic imbalances in chess. By assigning variable masses to pieces based on their influence and tracking their collective dynamics, this approach enables an objective measurement of power shifts throughout a game. The results suggest that the trajectory of the center of mass correlates with positional control, offering a novel macroscopic perspective on gameplay evolution. Although this methodology does not replace traditional move-by-move evaluation, it complements existing approaches by revealing overarching trends and transition points that are not easily discernible through conventional analysis. Future studies will focus on validating these findings through statistical correlations with known strategic paradigms, assessing the sensitivity of mass assignments, and refining the methodology to account for additional factors such as piece exchanges and king safety. This systematic approach opens new avenues for integrating physical principles into chess analysis, bridging strategic evaluation with quantitative methods.

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